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Research article

How do pre-service mathematics teachers resolve proportion tasks? Focus on the problem-solving strategy

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Abstrak Penyelesaian soal perbandingan belum sepenuhnya memasukkan berbagai strategi yang tepat. Penelitian ini bertujuan untuk mengidentifikasi dan menganalisis strategi yang digunakan calon guru matematika dalam menyelesaikan masalah perbandingan. Penelitian ini menggunakan metode kualitatif dengan desain fenomenologis. Partisipan penelitian ini berjumlah 29 mahasiswa calon guru matematika yang telah mempelajari konsep perbandingan. Data dikumpulkan menggunakan teknik tes, wawancara, observasi, dan studi dokumen. Data dianalisis secara bertahap, mulai dari pengumpulan data, reduksi data, penelaahan data, dan penarikan kesimpulan. Hasil yang diperoleh berupa uraian tentang teknik-teknik yang digunakan dalam menyelesaikan tugas terkait materi perbandingan. Mahasiswa calon guru matematika sebagian besar menggunakan strategi *cross-product* dalam menyelesaikan tugas perbandingan. Hasil penelitian ini dapat digunakan sebagai kerangka kerja untuk mengembangkan *hypothetical learning trajectory* dari materi perbandingan untuk mahasiswa calon guru matematika di masa yang akan datang.

Kata kunci *Calon guru matematika, Tugas perbandingan, Strategi pemecahan masalah*

Abstract Problem-solving for proportion tasks has not fully incorporated various appropriate strategies. This study aimed to identify and analyze the strategy used by pre-service mathematics teachers in solving proportion problems. This research used a qualitative method with a phenomenological design. The participants of this study were 29 pre-service mathematics teachers who had learned the concept of proportion. Data were collected using tests, interviews, observation, and document study techniques. Data were analyzed in the following stages: data collection, data reduction, data review, and conclusions. The results obtained were in the form of a description of the techniques used in solving proportion tasks. The pre-service mathematics teachers were mostly employing the cross-product strategy in solving proportion tasks. The results of this study can be used as a framework to develop the hypothetical learning trajectories of proportion material for pre-service mathematics teachers in the future.

Keywords *Pre-service mathematics teacher, Proportion task, Problem-solving strategy*

Introduction

Proportions are two equivalent ratios (Musser et al., 2014). The two ratios represent a comparison of relative values in different amounts. Proportion is the basis that will be used in discussing further mathematical topics (Bintara & Suhendra, 2021). Proportion is a fundamental skill that serves as the basis for advanced mathematical concepts such as algebra, calculus,

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geometry, and probability (Cox & Root, 2020). In addition to being a foundation in advanced mathematics, the concept of proportion also has an influence on developing students' knowledge and skills (Misnasanti et al., 2017).

The influence of students' knowledge and skills in solving proportion problems has a strong connection to daily life experiences and is important for achieving proficiency in more advanced mathematical ideas (Rosyidi & Hasanah, 2022). The ideas of ratio and proportion are frequently encountered by students in their daily lives, giving them a significant focus of study in mathematics education (Pratiwi & Sudihartinih, 2021). The significance of proportion in acquiring mathematical knowledge has been underscored, particularly within the framework of realistic mathematics education (Bintara et al., 2020). Understanding the concept of proportion requires proportional reasoning. Proportional reasoning is included in the component of mathematical ability, namely logical thinking and is an important thing to be mastered by students and teachers (Weiland et al., 2021). Proportional reasoning is important in understanding many situations in mathematics and in everyday life (Arican et al., 2018).

Proportion is a fundamental skill in mathematics education, serving as a bridge between basic arithmetic and more advanced mathematical concepts (Lutfi et al., 2022). Proportional involves a range of mathematical concepts, including fraction equivalency, division, place value, percentage calculations, and measurement conversions (Supply et al., 2023). It is considered crucial for students to develop a strong grasp of proportional reasoning as it forms the foundation for understanding and studying mathematical material (Lutfi et al., 2022). Researchers found that pupils had trouble understanding and drawing representations of quantitative relationships, especially those containing relational proportion (Permatasari et al., 2021). According to the studies, kids had a hard time with three things: answering questions, predicting patterns, and chunking data. In addition, proportionate reasoning includes the ability to manipulate verbal and arithmetic analogies, as proposed (Sari et al., 2023). Teachers are essential in facilitating diverse learning opportunities for proportional reasoning, including modifying the percentage bar model (Büscher, 2021). Moreover, the COVID-19 pandemic has had an impact on pupils' ability to reason proportionally in mathematics (Rohati et al., 2021).

Recent studies have demonstrated that students’ capacity to make well-informed financial choices is influenced when they employ additive techniques in proportional scenarios that necessitate comparisons (Scheibling-Sève et al., 2022). Furthermore, proportional reasoning is essential for higher-level math classes, providing access to higher education and employment opportunities (Cox & Root, 2020). Some of the math topics that require proportional reasoning include algebra, fractions, percentages, geometry, data graphing, and probability (Van de Walle et al., 2014). Seeing the number of mathematical topics that require proportional reasoning ability, Pyper (2014) suggests that this ability is one of the important factors in the development of individuals to understand mathematics. Based on this, mastery of proportional reasoning ability will also affect students' mathematical achievement.

Many studies are commonly investigate the students proportional reasoning (Arıcan, 2019; Ayan-Civak et al., 2023; Burgos et al., 2022; Diba & Prabawanto, 2019; Yanti et al., 2023), yet only few of them were concerned on the teachers and pre-service teachers proportional reasoning. The teachers’ proportional reasoning ability certainly affects the way they will transfer the mathematical knowledges to the students (Copur-Gencturk et al., 2022). In this case, the students’ mathematical understanding on proportion concepts is determined by the appropriate topics selected by the teachers. In line with that Dejene (2020) states that the things that a teacher considers to achieve his desired goals are the teacher's conception of teaching itself. The incompatibility between the beliefs and conceptions of teachers with the vision that directs the curriculum is one of the factors that hinder the implementation of the curriculum in accordance with the mission that has been set.

Several recent studies have shown that teachers have poor skills in solving proportion problems as well as experience the same difficulties as students (Burgos & Godino, 2022). The teachers only rely on strategies on calculation procedures, such as cross-multiplication and the use of incorrect strategies (Fernández et al., 2012). They do not focus on the fixed relationship between two quantities standing together, while also having difficulties to relate the two quantities in the proportion task (Arican, 2019b).

Regarding a direct and inverse in the proportion concepts, the teachers usually pay attention to qualitative relationships and the fixed rate of change (Arican, 2019a). The pre-service teacher students who were able to solve problems involving proportional situations were unable to provide an argument underlying the usage of their own strategies. Johar et al. (2018) asserted that the lack of proportional reasoning ability of pre-service teacher students is because they are accustomed to focusing and memorizing the steps to get the solution of a problem while they were at school level. Similarly, the study by Grace-Bridges (2019) shows that pre-service mathematics teachers are still having difficulty on the representations of ratios and providing the inappropriate arguments when explaining their solutions.

As for previous research that focused on pre-service teachers using 10 proportion tasks involving missing value problems, they are also struggling to argue the reason underlying the technique to solve non-proportional relationships and proportion tasks (Arican, 2019b; Izsák & Jacobson, 2017). A similar argument was presented by Joshua & Lee (2022) who concluded that the pre-service mathematics teachers employ only one strategy when solving proportion tasks. Based on the limitations and findings of previous research, we can propose a new problem context dimension that not only investigates the “what” and “how” but also explores the “why” and “when” aspects in selecting and implementing the appropriate strategy to solve proportion tasks. Despite the importance of proportion concepts in mathematics education research, the study on pre-service mathematics teachers’ knowledge and perceptions of proportion tasks is quite rare. Based on these facts, this study aims to identify the activities that pre-service teacher secondary school perform as well as the difficulties and the strategies they determine in solving proportion tasks.

Theoretical review

Proportion

Proportion is defined as a statement about the equality of two ratios (Johar et al., 2017). The concept of ratio has been defined differently by Euclid and Euler. In Euclid's book, the ratio is described as a relationship in terms of size between two equal quantities, where the quantities have ratios to each other that, when multiplied, can exceed each other (Grattan-Guinness, 2004; Simson, 1838). On the other hand, Euler in his book "Elements of Algebra" categorizes ratios into arithmetic ratio, which is the difference between two numbers, and geometric ratio, which represents how many times one number is greater than another number. Geometric ratios are obtained by dividing an antecedent by a consequent, and this concept involves considering the antecedent, consequent, and ratio resulting from the division (Williams, 2019). Proportion is the concept of equality between two ratios, enriched by historical interpretations ranging from Euclid's size relationship to Euler's distinctions between arithmetic and geometric ratios.

When discussing ratios, it is certainly inseparable from one of the most frequently used strategies to solve them, namely the "rule of three." The "Rule of three" can be expressed in modern algebraic notation as follows (Madden, 2018):

$$\text{For every positive number a,b,c,x if } \frac{a}{b} = \frac{c}{x} \text{ then } x = \frac{b.c}{a}$$

The ancient Babylonian artifacts provide evidence of the existence of the concept of ratio and proportion in ancient times. The artifacts exemplify the application of a rule to determine the value of x, given that a, b, and c are known variables in a transaction (Lin et al., 2020). For example, the price of one unit of an item is denoted as b silver coins. If c units of the item are being purchased, the total price is calculated by multiplying b and c, and then dividing by a. This ancient practice demonstrates the presence of the notion of ratio and proportion in ancient times, despite the fact that the people of that period may not have had explicit knowledge of this concept.

Contributions to mathematics from al-Khwarizmi's writings in Baghdad in the early 9th century, especially the "Treatise on Algebra," were substantial. By uniting Arithmetic and Geometry, his contributions not only advanced calculation but also established a brand-new branch of mathematics (Denning & Tedre, 2021). In order to distinguish between the various numerical roles, al-"Chapter of Transactions" Khwarizmi's work offers definitions of the Arabic terms quality evaluation, rate, price, and evaluated quantity. Al-Khwarizmi explained how to use the "rule of three" in the book "Treatise on Algebra" in the early 19th century in Baghdad. In the "Chapter of Transactions" there is the meaning of the words evaluation quality, rate, price and evaluated quantity which is a translation from Arabic used to distinguish the role of the numbers used. The writing if notated in mathematics as follows:

$$\frac{quality\ evaluation}{rate} = \frac{evaluated\ quantity}{price}$$

Mathematically the ratio is written:

$$a{:}b\ or\ \frac{a}{b}\ with\ b \neq 0$$

The ability to apply mathematical principles in various circumstances is evident here, laying the groundwork for future advancements and developments in the field (Denning & Tedre, 2021). The main idea of ratio comes from thinking of the concepts of decimal numbers, fractions, and percents in a more modern view where basic ideas such as partitioning and division are more involved. Equivalence, often seen as sameness, can be defined as a similarity, or often referred to as proportion. Some applications of scale in photographs and maps.

Furthermore, the topic of proportion first entered the school mathematics curriculum in the 12th century AD (Høyrup, 2005). Proportion is a concept that appears in mathematics and physical science to describe the relationship between two quantities or quantities. (Borowski & Borwein, 1989)explain that proportion is a linear relationship between two variable quantities, or their inverses, where the corresponding elements of the two proportional sets have a constant ratio. This means that if we have two sets of data that have a proportional relationship, the ratio between the matching elements will always remain the same.

Proportionality, in a more detailed way, explains that when two quantities, x and y, are connected through the equation y = kx, where k is a constant of proportionality, then y is said to be directly proportional to x (Clapham & Nicholson, 2009). This means that the quantity y will change in proportion to the change in quantity x. Conversely, if the equation is y = k/x, then y is said to be inversely proportional to x, meaning that as x increases, y will decrease inversely. More simply, this proportional relationship can be represented as a linear function y=kx for equal proportions or $y = \frac{1}{k}x$ for inverse proportions (Lamon, 2020). In addition to being represented as a linear function, proportional problems can also be interpreted as problems involving two equivalent ratios (Ben-Chaim et al., 2012). Ratio is a term that refers to the comparison of two or more quantities (Lamon, 2020). The two ratios present a comparison of relative values in different amounts(Musser et al., 2014). Mathematically it can be defined:

$$\text{For example, } \frac{a}{b} \text{ and } \frac{c}{d} \text{ are two ratios}$$

$$\text{Then } \frac{a}{b} = \frac{c}{d} \text{ if and only if } ad = bc$$

When four quantities, a, b, c and d, have proportional magnitudes such that $a/b= c/d$, they are called proportional. This relationship is often expressed as a is to b as c is to d. In notation that is no longer commonly used, i.e., in the obsolete notation $a : b \equiv c : d$. The terms a and d are known as the extremes, while b and c are referred to as the means in this context. The terms a and d are known as extremes, while b and c are referred to as averages in this context. Componendo, dividendo, and convertendo are operations in mathematics that can be used in proportion equations to produce useful forms. The concept of proportion is important in understanding and solving problems involving quantities that are comparable to each other.

There are two types of proportions, namely direct proportion and inverse proportion (Nicholson, 2014). Equivalent comparison is defined as if two quantities x and y are connected in an equation y=kx, where k is a constant, then y is called equivalent comparison with x, it can be written y~x. The constant k is the proportionality constant. It can also be said that changes in the y value are directly proportional to changes in the x value. If y is plotted based on the x value, a line graph will be formed. An inverse proportion is defined if y=k/x, then y is inversely proportional to x. It can be written as y ~ 1/x, and it can be said that the change in the value of y will be inversely proportional to the change in the value of x. A like-for-like comparison between two quantities occurs when quantitative changes occur to both equally. In other words, if a quantity a is multiplied by a factor m, then quantity b must be multiplied by m which is a constant factor. In this case, the ratio between the first two quantities is equal to the ratio of the second pair of quantities.

Strategy for solving proportion tasks

Proportion problems can be solved through various strategies, including unit rate strategy, factor of change strategy, fraction strategy, building-up strategy, and cross-product strategy (Arican, 2018; Tunç, 2020). The categories of informal strategies include unit rate strategy, factor of change strategy, fraction strategy, and building-up strategy. One of the most commonly used informal strategies is the building-up strategy. In this strategy, one establishes a ratio and extends it to other ratios by using repeated addition (Lamon, 2020). Children spontaneously use intuitive strategies. Furthermore, researchers have observed that students can solve proportion issues without explicit instruction, indicating the development of intuitive strategies for handling proportions (Kent, 2017). While the formal strategy category uses an equation-based approach (cross-product) as a solution to the highest level of proportional problem solving, If a proportion problem involves a missing value, both formal and informal strategies can be used for problem solving, as shown in Figure 1.

The price of 4 kg of apples is Rp20,000. What is the price paid for 18 kg of apples?

Figure 1. Example of missing value problem

The first informal solution to the unit rate strategy involves determining the value per unit of a quantity and applying it to calculate comparable values for other units. Research conducted by Jayasuriya et al. (2019) has shown that this strategy effectively improves students' understanding of proportional relationships and their ability to solve practical problems related to ratios and rates. The use of the unit rate strategy is for students to first find the value of one unit. Once this value is found, the student can calculate the actual value requested. An example of a solution to the problem presented in Figure 1 with the unit rate strategy is to first calculate the price for 1 kg of apples, then multiply the price of 1 kg of apples by 18 kg.

The second informal strategy, the factor of change strategy, helps students determine the ratio of changes between one quantity and another in a proportional relationship. The strategy can be used to estimate the value of a quantity based on changes in other quantities (Weiland et al., 2021). The use of the factor of change strategy leads students to find the multiplier factor, which is then applied to determine the requested value. An example solution to the problem presented in Figure 1 is to find the relationship between 4 kg and 18 kg. It turns out that 18 kg $= \frac{9}{2} \times 4$ kg. This means that $\frac{9}{2}$ is the multiplier, so that if 4 kg costs Rp 120.000, then for 18 kg the price is $\frac{9}{2} \times 120.000 = Rp\ 540.000$,-.

The third informal strategy, the fraction strategy, utilizes fractions to represent proportional relationships and solve proportional problems by applying fraction properties. Previous research Gabriel et al. (2013) has emphasized the significance of this strategy in enhancing students' understanding of proportional reasoning and their ability to solve complex proportion problems involving fractions. The use of the fraction strategy leads students to consider the ratio formed as a fraction, so that to determine the requested value, students only need to apply the fraction equivalence rule, which is multiplying or dividing the numerator and denominator by the same number. To find a solution to the problem presented in Figure 1, the first step is to form a ratio that expresses the comparison between the known values, specifically the comparison between apples and prices. The fraction formed is $\frac{4}{120.000}$. Furthermore, to find the price of 18 kg of apples, it can be done by multiplying or dividing the numerator and denominator by the same number until the numerator reaches 18. When the numerator has reached 18, the number contained in the denominator is the asking price.

The fourth most common informal building-up strategy involves the gradual construction or magnification of one quantity to achieve a proportional relationship with another quantity. This strategy assists students in understanding the concept of proportion and enhances their ability to solve problems involving the construction or proportional magnification of quantities (Yu et al., 2020). Building-up strategies are often known as intuitive strategies. Students dabble, simulate, list, and look for patterns. Example the problem presented in Figure 1 can be solved with the building-up strategy as follows:

$$4\ kg \longrightarrow Rp\ 120.000,\text{-}$$
$$18kg = 4kg + 4\ kg + 4kg + 4kg + 2kg$$

then the price of 18 kg:

$$120.000 + 120.000 + 120.000 + 120.000 + 60.000 = 540.000$$

The formal solution is the cross-product strategy, a formal strategy that uses a standard algorithm that involves creating an equation for two ratios, one of which has an unknown quantity, cross-multiplication, and solving the equation for the unknown quantity (Tunç, 2020). The cross-product strategy is a common approach used in solving proportional problems by multiplying the product of the corresponding components of two proportional quantities. Research shows that teachers often use formal strategies, such as assigning proportions, to solve proportional reasoning problems (Brakoniecki et al., 2021). So, the cross-product strategy is the most commonly used strategy because of its highly efficient process. Based on this, the solution to the problem presented in Figure 2 is:

$$\frac{4}{18} = \frac{120.000}{x} \Leftrightarrow 4x = 2.160.000 \Leftrightarrow x = 540.000$$

So in this research the strategies we will investigate are unit rate strategy, factor of change strategy, fraction strategy, building-up strategy, and cross-product strategy in solving proportion problems. The application of these strategies can be used as an alternative to support the performance of students, prospective mathematics teachers, and mathematics teachers in solving problems related to proportional reasoning. The strategies used also provide insight into the diverse approaches students use to solve proportion problems (Karli & Yildiz, 2022). Thus, it can be concluded that proportion is the relationship between two or more equivalent ratios. The relationship in this proportion problem can be divided into two types, namely equal direct proportion and inverse proportion.

Methods

This research uses a qualitative approach which aims at in-depth understanding of the phenomena and exploring individual perspectives and experiences in social, cultural, and historical contexts (Creswell, 2017). The type of qualitative design used is phenomenology. Phenomenological research design aims to explore a subject by conducting interviews or observations to order to gain a detailed understanding of the phenomenon under study (Gill, 2020). The phenomenon to be investigated in this study is related to the proportion assignment work given to the pre-service mathematics teachers (PMT) of the university in Indonesia.

The participants in this study were 29 pre-service mathematics teachers (PMT). The participants were in the second year of their program in Mathematics Education Department.

The selection process of participants was based on the consideration that the PMT students had studied the proportion content with the aim of being the main source for gathering the data. The proportion concept is part of school mathematics curriculum and one of the early algebra topics. Many proportion problems can be solved through various solution strategies. The research place was determined with consideration to facilitate the research process because it was in the same location as the researcher. All participants first read about the rules of the research to be carried out. In addition, participants were willing to be participants in this study. This research was conducted in accordance with the Helsinki guidelines, and the protocol was approved by the Ethics Committee of Universitas Pendidikan Indonesia with the number 6003/UN40.F4.D1/KM/2023.

The instruments used in this study include proportion test items, interviews, and documentation, which were designed to collect the data of the strategies used by the PMTs in solving proportion problems. The proportion test, consisting of two items. The first item of the test explores the strategies used by the participants in solving direct proportion task, while the second test item aimed to uncover how they were dealing with inverse proportion problems (Tabel 1). To complement the data from the test, interviews were conducted to further explore the PMT understanding and strategies. The interview aimed to explore PMT understanding of proportion concepts, the approaches they have employed, and the rationale behind choosing certain strategies. The interview questions included: (1) what strategy did you use in solving the problem?; (2) why did you use that strategy?; and (3) what strategies do you know in solving comparison problems?

These questions were designed to not only identify the strategies used but also to uncover any difficulties or obstacles PMT may have encountered. In addition, documentation, which includes all forms of written or electronic recordings related to the research, including but not limited to, test recordings, interview notes, and PMT worksheets, played an important role in providing empirical evidence supporting the research findings and allowing for more in-depth data analysis. The combination of these methods proportion tests, interviews, and documentation enabled the researcher to collect comprehensive data on the strategies used by the PMT in solving proportion problems.

Table 1. Test items for proportion concept

No.	Task
1.	To travel 125 kilometers, a car needs 10 liters of fuel. If there is still 10 liters of fuel in the tank and the car will travel 300 kilometers, then the minimum fuel that must be added? try to solve it in 2 ways!
2.	Mr. Ujang will build a house. He estimates that the house will be completed in 15 days by 8 workers. The workers start working at 7 am and finish at 5 pm every day. For some reason, Mr. Ujang wants to finish the house 5 days faster. Therefore, he added 8 more workers. How many hours do the workers work each day so that the work is completed on time?

This research outlines a data analysis process that is carried out systematically, including data collection, data reduction, data analysis, and conclusion. The data collected came from tests implemented through PMT worksheets, focusing on proportion-related tasks. In the data reduction phase, a selection was made of the documents based on the PMT answers, where the answers were classified according to the characteristics of the strategies used. Data analysis involved an examination of the characteristics of the solution strategies used in the PMT, coupled with interviews with a number of PMT who applied various strategies. Conclusions were drawn by identifying the characteristics of strategies frequently used by PMT. To support the data analysis process, this study used the qualitative software, Atlas.ti, which facilitates the researcher in managing and categorizing the PMTs' responses based on Table 2.

Table 2. Characteristics of proportion strategy

Strategy	Characteristics
Cross-product	The students' responses indicate a solution for proportion tasks through the usage of a variable and the implementation of rule of cross-product. The students' responses indicate the existence of relationship between covariance and invariance.
Unit rate	The students' responses indicate a determination of the value per unit of a quantity and comparable values for other units.
Factor of change	The students' responses indicate a determination of multiplier factor, which is then applied to determine the required value.
Fraction	The students' responses indicate the use of mathematical representation to the problem in fraction form and a determination of the required value through the implementation of the rule of equivalent fractions.
Building-up strategy	The students' responses involve gradually building up or enlarging both measures of one quantity to achieve a proportional relationship with the other quantity. The students' responses indicate the usage of pictures or mathematical models that compatible with the problem.

Findings and Discussion

Task design for direct proportion

In the first problem situation, as shown in Figure 2, in the assignment given on comparative value in class, the preservice teacher had to solve the task with two strategies with the distance value at a known car speed and the fuel required for a certain distance.

To travel 125 kilometers, a car needs 10 liters of fuel. If there is still 10 liters of fuel in the tank and the car will travel 300 kilometers, then the minimum fuel that must be added? try to solve it in 2 ways!

Figure 2. Task design for direct proportion

In the interpretation of the technique for solving proportion tasks, in the first direct proportion task situation, PMTs used the cross-product strategy by determining two ratios: from the distance at a known fixed car speed to the fuel required at a certain distance, and then solving the equation for the unknown quantity as shown in Figure 2. Almost half of the PMTs, specifically 10 PMTs, used the cross-product strategy to solve direct proportion tasks related to car fuel requirements. For example, M16 in Figure 3 completed determining the ratio of distance to car fuel, $\frac{125}{10} = \frac{300}{x}$, then $125x = 3000$ is done by cross-multiplication and $x = \frac{3000}{125} = 24$ liters is obtained, the minimum fuel that must be added is $24 - 10 = 14$ liters, which answers correctly, while 5 PMTs answer incorrectly. For example, M20's basic arithmetic error in Figure

3 is $125 = 10l$, $300 = x$, cross-multiplication is obtained $125x = 3000$, so $x = \frac{3000}{125} = 19.5$, which is the result of an incorrect answer, which results in an incorrect final result of $19.5 - 10 = 9.5$, which should be $24 - 10 = 14$. PMT errors were also found in M4, M23, M9, and M18, which could not determine the quantity relationship due to the limitations of the previous ratio concept.

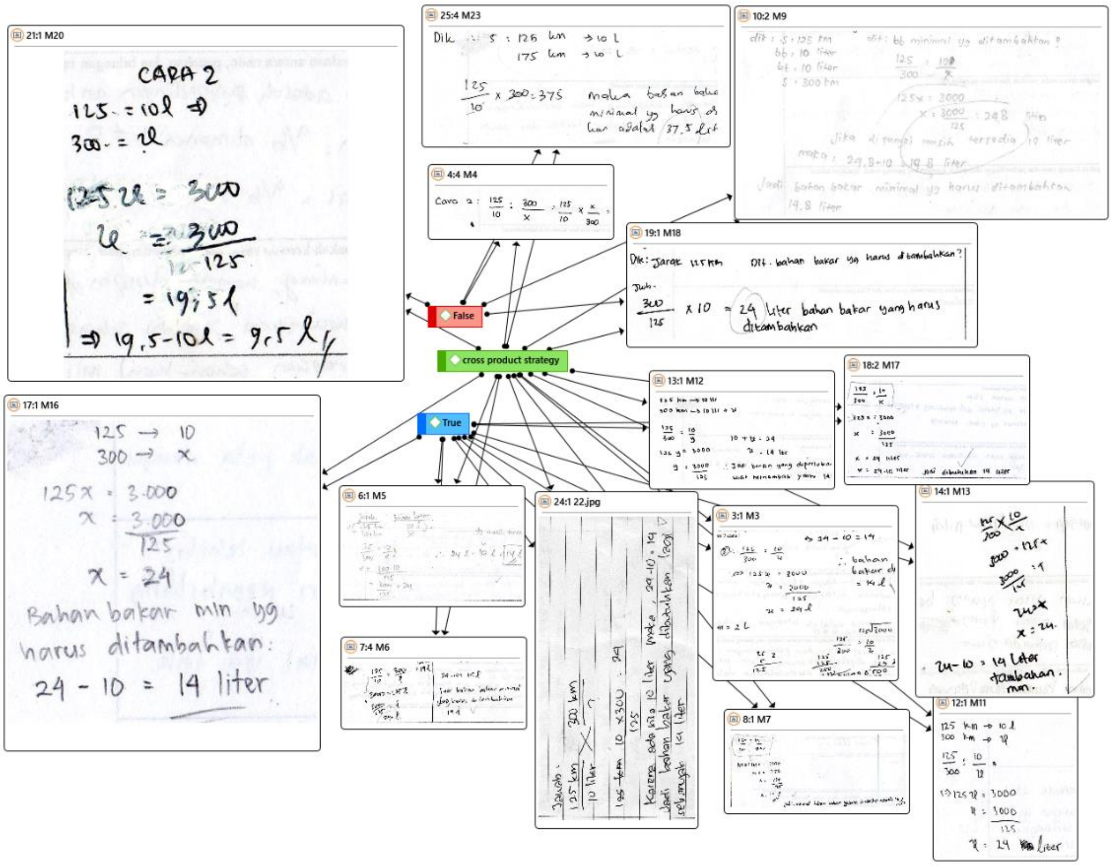

Figure 3. PMTs' responses to direct proportion task using cross-product strategy

The second direct proportion task situation was solved using the unit rate strategy by 16 PMTs. In this strategy, 10 PMTs can answer correctly and 6 can answer incorrectly. For example, M16 in Figure 4 completed the direct proportion by determining one unit, $\frac{125}{10} = 12.5$, which is 1 liter of fuel enough for 12.5 kilometers. So, to determine gasoline fuel for 300 kilometers by dividing 300 by one unit, namely 12.5, the result is 24 liters. The completion of M16 the first way in Figure 4 shows understanding by using the cross-product strategy, which is the second way of solving the same result. However, in PMT M24 in Figure 4, the completion can determine one unit of 125 kilometers divided by 10 liters to 12.5. However, there is confusion in the solution, namely $300 - 10 = 175$, $\frac{12.5}{175} = 134$. In conclusion, the fuel added is 134. PMT determines the difference in distance with gasoline fuel with the wrong calculation as well, and the impact of the answer also determines the wrong and unclear steps. In this strategy, accuracy is needed in completing the task of each unit as determined by the appropriate multiplier.

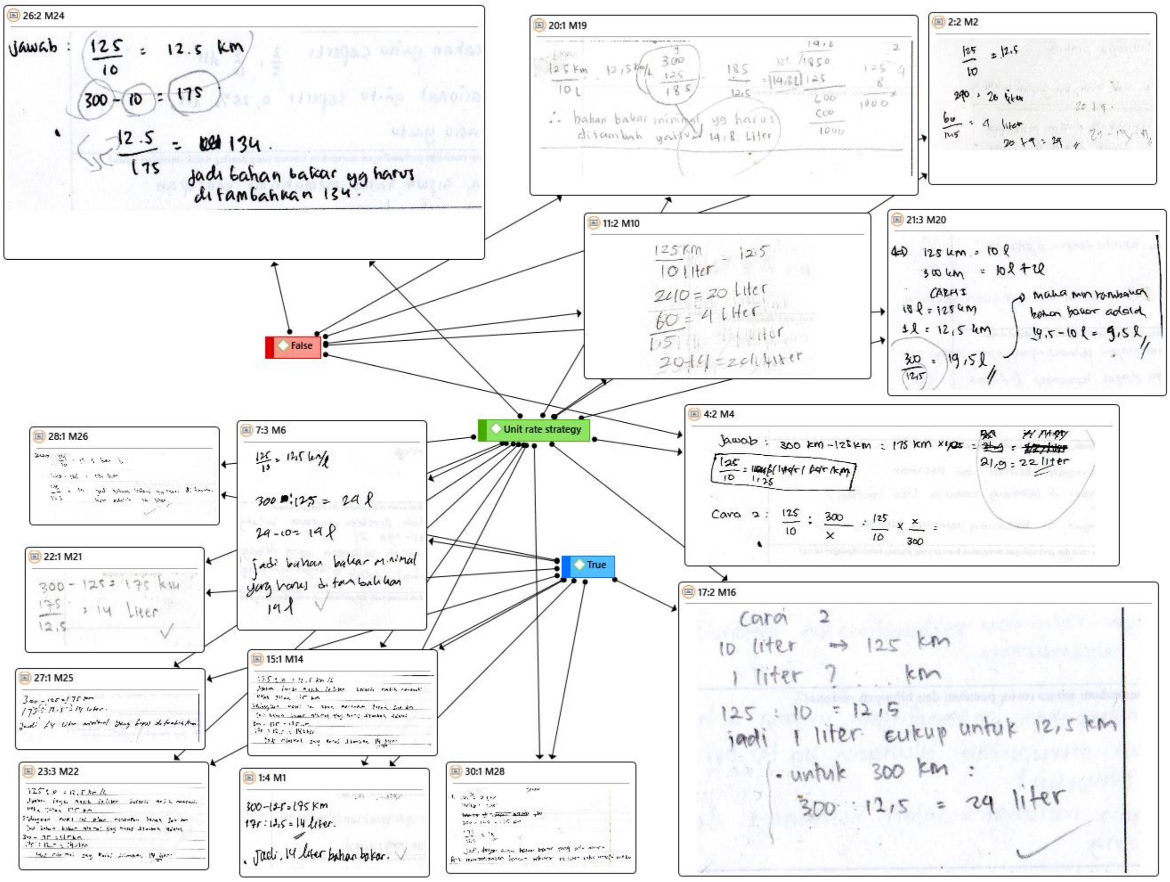

Figure 4. PMTs' responses to direct proportion task using unit rate strategy

The third direct proportion task situation using fraction strategy is the student's answer by representing the problem in the form of fractions and determining the required value by applying the rule of equivalent fractions, but there are no students who can use the strategy with fractions. For example, M8 in Figure 5 forms the distance problem with fuel consumption with $\frac{125}{10} \times 100$ and $\frac{300}{10} \times 100$ without giving the final answer correctly, while M19 writes $\frac{125}{10}$ with $\frac{185}{12.5}$ with the wrong answer (see Figure 5).

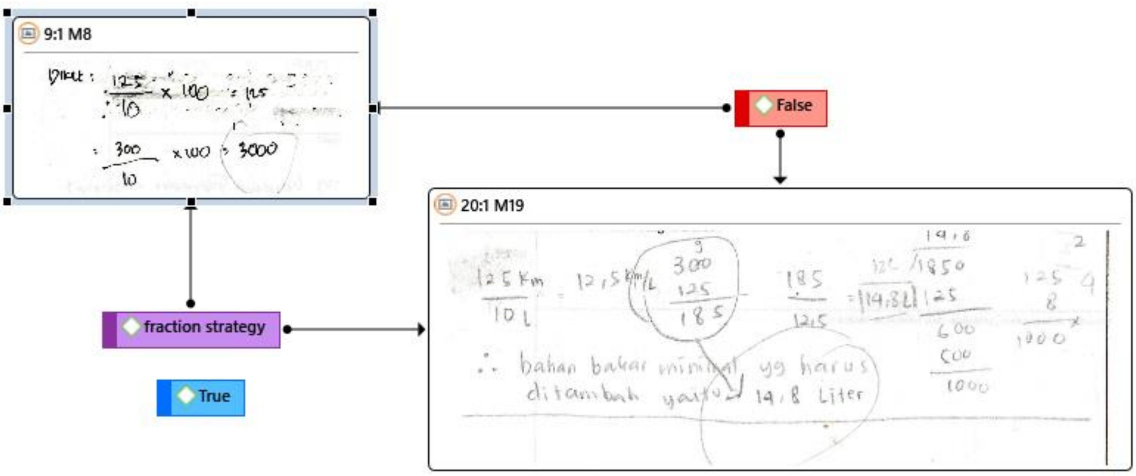

Figure 5. PMTs' responses to direct proportion task using fraction strategy

The fourth direct proportion task situation using the equation strategy is using variables and solving variables in the model in general. However, as exemplified by PMT code M27 with the solution $125 = 10x, x = \frac{125}{10} = 12.5$ by making the same equation relationship $300 = y\ (12.5)$ so that $y = 300.\ 12.5 = 36$ liters, which is then 24 minus 10 liters, so that the remaining fuel needed is 14 liters (see Figure 6). The wrong answer should be $y = \frac{300}{12.5}$, which results in 24.

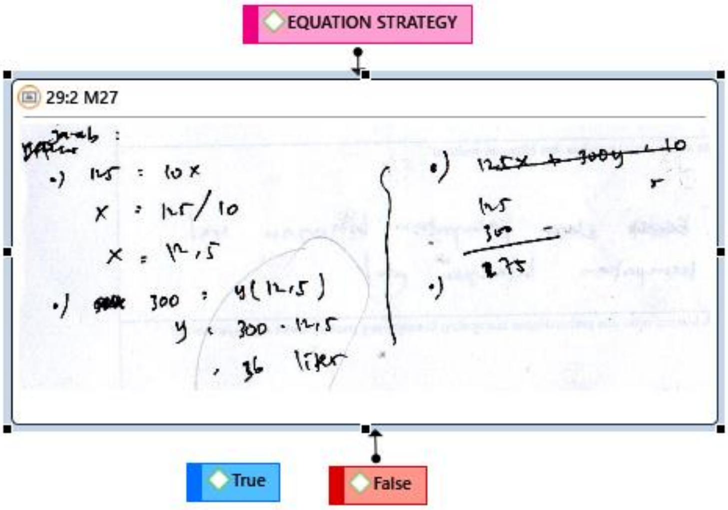

Figure 6. PMTs' responses to direct proportion task using equation strategy

The fifth direct proportion task situation using the Building Up strategy is the student's answer by gradually building or enlarging both sizes of one quantity to achieve a proportional relationship with the other quantity. The PMT answer is built starting from every 125 kilometers = 10 liters, so that every 25 kilometers = 2 liters, then 50 kilometers = 4 liters, so that the PMT constructs the proportion of 125 kilometers = 10 liters, 250 kilometers = 20 liters, so that when 300 kilometers = 250 kilometers + 50 kilometers = 20 liters + 4 liters = 24 liters, as exemplified by the PMT code M3 with the correct answer (see Figure 7).

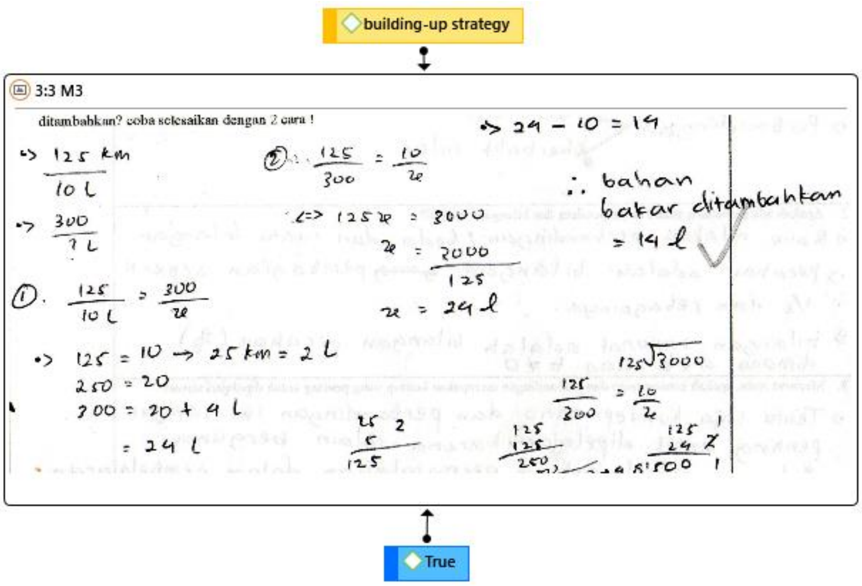

Figure 7. PMTs' responses to direct proportion task using building-up strategy

In the context of Task Design for Direct Proportion, various strategies can be utilized, including the unit rate strategy, factor of change strategy, fraction strategy, building-up strategy, and cross-product strategy (Lamon, 2020). Within this scenario, five strategies were identified among PMT. Primarily, the cross-product strategy was preferred by many due to its efficiency. PMT Teachers typically initiate the process by forming two equivalent ratios, followed by a cross-multiplication procedure, and concluding with a division procedure. Most of the PMTs used the cross-product strategy to solve direct proportion problems. Based on an interview with one of the PMTs who explained that he used the cross-product strategy, the following is his statement:

Q : What strategy did you use to solve the problem?
M13 : The cross-product method.
Q : Why did you use that strategy?
M13 : I learned the cross-product method at school.
Q : What strategies do you know for solving comparison problems?
M13 : There are many, but I only know the cross-product method.

PMT explained that the cross-product strategy is often used because of the experience she gained when being taught by teachers at school. Second, the Unite Rate Strategy was also the choice of PMT in solving the Direct Proportion Design Task by finding the value of one unit first and then determining the specified value. Based on an interview with one of the PMTs who explained that he uses the unit rate strategy, the following is his expression:

Q : What strategy did you use to solve the problem?
M10 : I started by determining one unit first, and then I multiplied by that unit.
Q : Why did you use that strategy?
M10 : It's easier.
Q : What strategies do you know for solving comparison problems?
M10 : comparison formula.

This unit rate strategy is the choice of some PMTs because it is easy to determine other larger units. The third PMT attempted to use the Fraction Strategy but was not precise in determining the rule for fraction equality, which involves multiplying or dividing both the numerator and denominator by the same number. Meanwhile, the fourth PMT tried using the Equation Strategy, treating it as a linear function $y = kx$. However, this PMT was unable to correctly identify the x and y variables, and therefore could not solve the problem correctly. Below is an interview with the PMT who used the Equation Strategy:

Q : What strategy did you use to solve the problem?
M27 : I used the equation, but it was wrong to determine the variable.
Q : Why did you use that strategy?
M27 : I remembered the equation.
Q : What strategies do you know for solving comparison problems?
M27 : I use equations, but sometimes I use comparisons.

Fifth PST tried with the building up strategy, namely trying to find a pattern of 125 km it takes 10 liters, meaning that every 25 km takes 2 liters, so the multiples are 125 km +125 km it takes 10 liters + 10 liters. Then for 300 km = 125 +125 + 25(2) it takes 10 liters +10 liters + 4 liters. So 300 km requires 24 liters. Below is an interview with the PMT who used the building up strategy:

Q : What strategy did you use in solving the problem?
M3 : just try it
Q : Why did you use that strategy?
M3 : Using logic, it is easier to determine what is asked.
Q : What strategies do you know in solving comparison problems?
M3 : using colon, a:b

The Direct Proportion Design task can use the technique of a ratio table interpreted on a graph but this was not used by PMT. PMT can create a ratio table from what is known and what is asked as in Table 3.

Table 3. Ratio table

Distance (km)	125	300
Gasoline quantity (Litre)	10	….

The factor of change strategy leads students to find the multiplying factor which is then applied to determine the required value. Based on this, the solution to the problem presented in Figure 1 is to find the relationship of 125 km and 300 km. It turns out that $300\text{ km} = \frac{12}{5} \times 125\text{ km}$. This means that $\frac{12}{5}$ is the multiplier factor, so if 125 km needs 10 liters of gasoline then for 300 km the need is $\frac{12}{5} \times 125\text{ km} = 24\text{ liters}$. Based on the results of answers and interviews with PMT, it was revealed why PMT solve direct proportion problems using various strategies. Most of them, in solving proportion problems, are influenced by learning experiences that affect the solution process. PMT mostly used the cross-product strategy as the main solution in solving it. In addition to the variety of strategies found by PMT, there are also errors or difficulties experienced in working on direct proportion problems.

The difficulties of PMT in completing Direct Proportion tasks are found in the confusion in basic arithmetic, non-patterned relationships, adaptive strategies in solving difference problems, and experimentation with equation strategies among PMT. The findings in these errors align with the research of (Joshua & Lee, 2022). Common errors that arise cannot be considered as simple unit or rounding errors. More precisely, these errors reflect issues in how respondents think about quantity, word problems, and the nature of mathematics itself. Arican, M. (2019) also found that prospective teachers have difficulties in representing and interpreting proportional and non-proportional relationships, with a focus on solutions or procedural steps. The research also highlights difficulties in understanding the requirements of statements, the context of problem situations, and the mathematical procedures involved in task resolution, as found by Burgos & Godino (2020). Such difficulties are also manifested in the challenges of solving irregular number problems, as expressed by Arican & Özçakir (2021).

The issues related to direct proportion can be presented through various strategies, including unit price strategy, change factor strategy, fraction strategy, development strategy, and cross-product strategy. Equivalence strategies, ratio tables, and graphs are also part of the strategies. PMT students were found to use the cross-product strategy as one of their main strategies. According to Son & Lee (2021) research, the solution strategies applied by prospective teachers and the determination and representation of relationships are influenced by the context of the problem. They indicate that prospective teachers excel in determining direct relationships, in line with the findings of Arican, M. (2020). The difficulties faced by these respondents can enhance problem solving through the use of proportional formulas, such as the cross-multiplication strategy, without having to interpret words or answers correctly, as revealed by (Cabero-Almenara et al., 2020). Thus, these findings overall provide insights into the challenges faced by prospective teachers in understanding and solving direct proportion tasks. These strategies serve as alternatives for PMT to enhance their professional competence in the future when completing direct proportion tasks.

Task design for inverse proportion

In the second problem situation, as seen in Figure 8, in the task assigned to the inverse ratio value, pre-service mathematics teacher had to complete the task of estimating the number of workers to build a house with a previously known completion time with a predetermined number of workers.

> Mr. Ujang will build a house. He estimates that the house will be completed in 15 days by 8 workers. The workers start working at 7 am and finish at 5 pm every day. For some reason, Mr. Ujang wants to finish the house 5 days faster. Therefore, he added 8 more workers. How many hours do the workers work each day so that the work is completed on time?

Figure 8. Task design for inverse proportion

In the interpretation of what is requested in the technique of solving the proportion task, in the first inverse proportion task situation using the cross-product strategy, as shown in Figure 9, PMT is considering the inverse proportion task based on the comparison of the length of time of workers with the number of workers needed to solve half using the cross-product strategy, namely by determining two ratios from the length of time of workers with the same number of workers. So that one of the two ratios has an unknown quantity with the variable x, perform cross-multiplication, and solve the equation for the unknown quantity as shown in Figure 9. This cross-product strategy requires only one person to be able to solve it correctly. For example, M5 in Figure 9 determines the relationship between the length of worker time and the number of workers, namely $\frac{15.8}{10.16} = \frac{x}{10}$; $\frac{120}{160} = \frac{x}{10}$. The cross-product is $1200 = 160x$, and then $x = \frac{1200}{160} =$ 7.5 hours. While 12 PMTs have not answered correctly using the cross-product strategy, for example, M4 completed with.

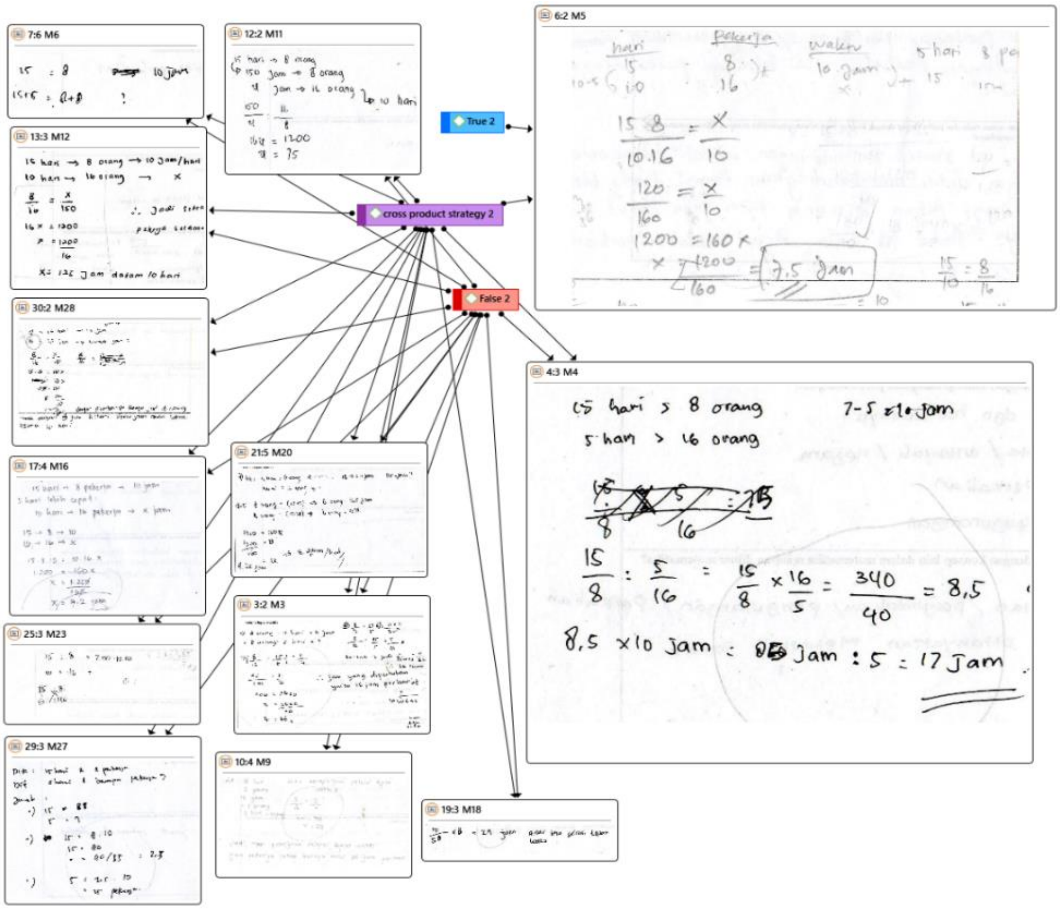

Figure 9. PMTs’ responses to inverse proportion task using cross-product strategy

Both groups of PMT solved inverse proportion problems using fraction strategies, but with different errors (see Figure 10). M26 was mistaken because they believed $\frac{15}{8}$ was equivalent to $7:8$ and $\frac{10}{16}$ was equivalent to $\frac{1}{1.6}$ without understanding the meaning of solving fractions. M24 was also wrong because they attempted to form ratios with fractions such as $\frac{15}{8} = 7$ and $\frac{10}{16} = 1.6$ without providing adequate explanation for the proportion solution. Meanwhile, M21 erred in determining the relationship by calculating $\frac{150}{8}$ as $\frac{50}{8}$, then subtracting it to become $\frac{100}{8}$, interpreted as 12.5 hours. M22 also erred by calculating $\frac{150}{8} = \frac{?}{16}$, then concluding that $\frac{8}{16}$ was equal to $\frac{1}{2}$, and $\frac{150}{300}$ was equal to $\frac{1}{2}$. M19 made a mistake by trying to answer how many hours 16 workers needed in 10 days, using incorrect calculations like $\frac{15}{8} \times 10 = 18.75$ and $\frac{10}{16} \times 18.75 = 11.71$, concluding that each worker required 11.71 hours for every 10 days. On the other hand, M8 was incorrect because they calculated $\frac{15}{8} \times \frac{7}{5}$ as $\frac{75 \times 56}{40}$, resulting in $\frac{4200}{40}$, which was concluded as 15.

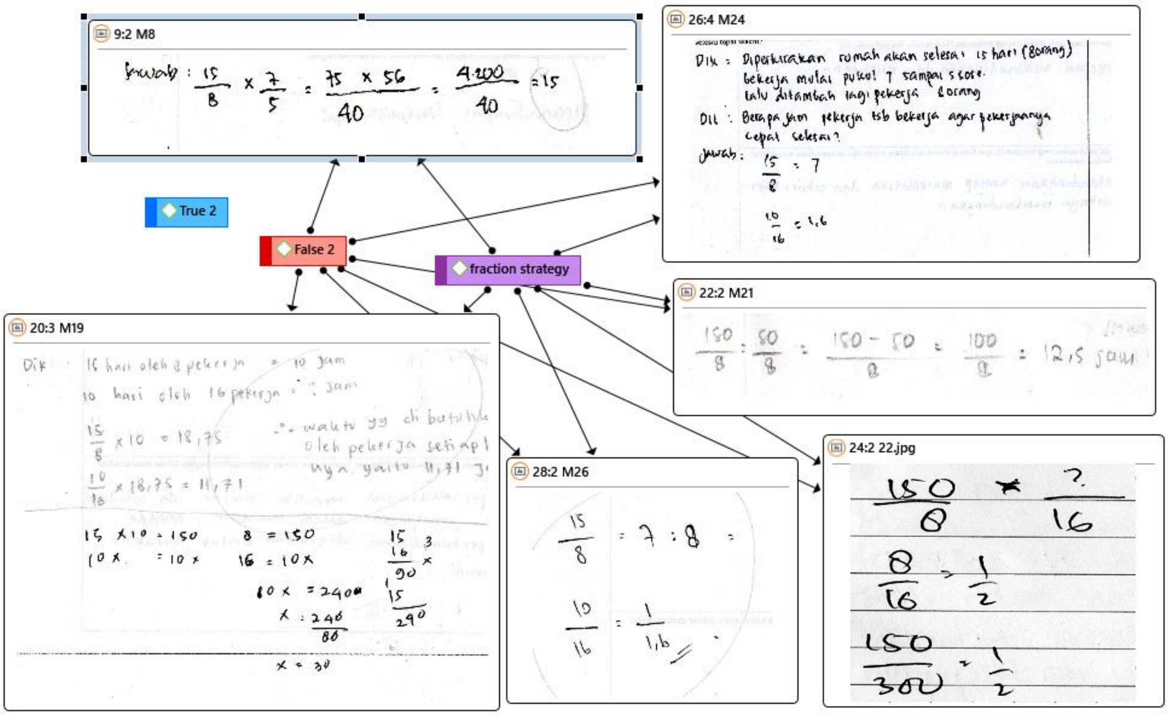

Figure 10. PMTs' responses to inverse proportion task using fraction strategy

In the third scenario of an inverse proportion task, illustrated by PMT code M22 and depicted in Figure 11, the factor of change strategy was employed. However, the PMT encountered an error in determining the multiplier factor, interpreting $15 + 8$ as $\frac{23}{7}$, and calculating $\frac{3.2+5}{5} + 8$ as 9.5 hours. This highlights the challenges faced by pre-service mathematics teachers in designing tasks and applying appropriate strategies for solving inverse proportion problems.

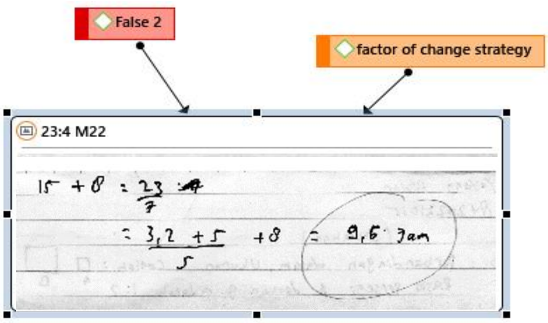

Figure 11. PMTs' responses to inverse proportion task using factor of change strategy

In the four scenarios of inverse proportion tasks, illustrated by PMT codes M1, M2, and M25 and depicted in Figure 12, the building-up strategy was utilized. However, all three PMTs provided incorrect answers. M1 applied the building-up strategy inaccurately, stating that given 15 days for 8 workers, then for 5 days it would be $8 + 8 = 16$ workers, resulting in 7 by 5 = 10 hours, and subsequently concluding that $10 + 10 = 20$ hours were needed. M2 attempted the building-up strategy by writing $8 \times 15 = \ldots$ but failed to complete the solution. Similarly, M25 employed the building-up strategy, knowing that if 15 equals 8 people, which equals 10 hours, then what about 16 people and 5 days? They attempted to deduce it gradually by considering the difference, starting from $15 - 5 = 10$ days, concluding that the workers must work for 20 hours per day. These instances underscore the challenge faced by pre-service mathematics teachers in effectively applying strategies to solve inverse proportion problems.

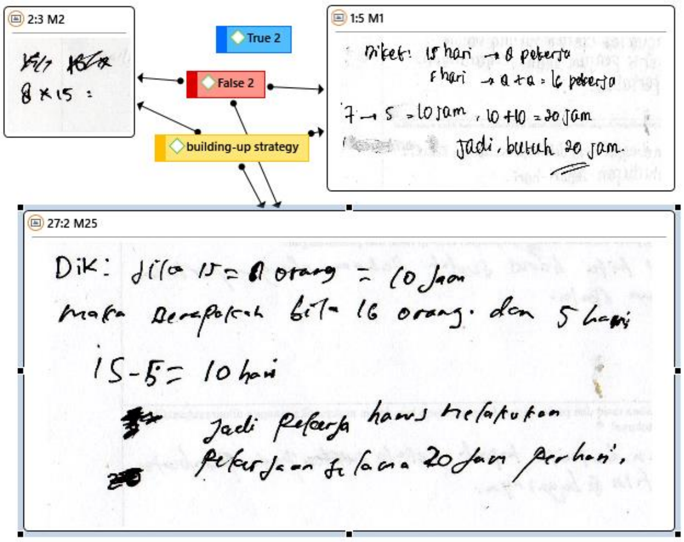

Figure 12. PMTs' responses to inverse proportion task using the building-up strategy

In the fifth inverse proportion task, exemplified by pre-service mathematics teacher code M13, the Two Variable Linear Equation Strategy was employed, albeit with an incorrect answer (see Figure 13). M13 attempted to solve the problem by equating 15 days to 8 workers to 8 hours, then defining the worker variable as x and the hour variable as y. This led to the equations $8x + 10y = 15$ and $16x + y = 20$, which were solved using the elimination method. Subtracting the second equation from the first yielded $128x + 160y = 200$, and $128x + 8y = 180$, resulting in $152y = 80$, and consequently $y = \frac{80}{152} = 0.52$. However, the solution provided by M13 was incomplete and inaccurate in determining the equations.

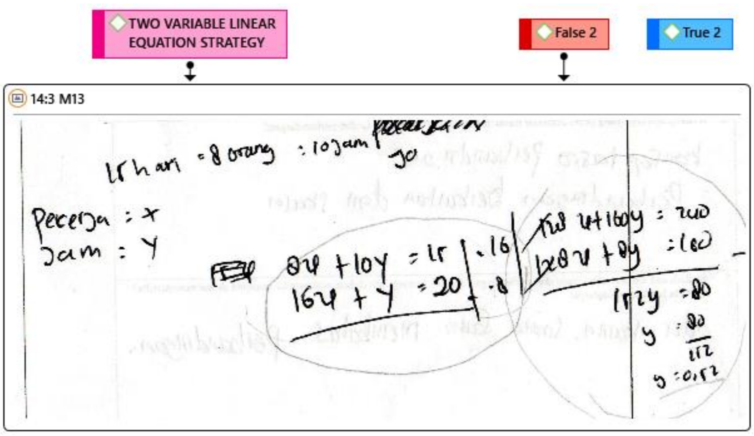

Figure 13. PMTs' responses to inverse proportion task using two-variables linear equation strategy

In the Task Design Inverse Proportion situation, there are 5 strategies found in pre-service mathematics teacher. First, more people choose the cross-product strategy because the process is very efficient. Based on an interview with one of the PMT who answered the inverse proportion task using the cross-production strategy, he revealed the following:

Q : What strategy do you use to solve the problem?
M5 : Cross-product.
Q : Why did you use that strategy?
M5 : It's easier to work with.
Q : What other strategies do you know for solving comparison problems?
M5 : I don't know any other strategies.

PMT starts with the formation of two equivalent ratios, then a cross-multiplication procedure is carried out, and finally a division procedure is carried out. For example, if the two equivalent ratios are expressed as $\frac{125}{10} = \frac{300}{x}$ then the cross-product procedure $125x = 3000$ then $x = \frac{3000}{125}$ = 24 liter. The second Unite Rate Strategy was also PMT choice in completing the Design inverse Proportion Task by finding the value of one unit first and then determining the specified value. For example $\frac{125}{10} = 12{,}5$ liter then $\frac{300}{12{,}5} = 24$ liter. The third PMT tried the Fraction Strategy but was incorrect in determining the fraction equivalence rule, which is multiplying or dividing the numerator and denominator by the same number. Based on an interview with one of the PMT who answered the inverse proportion task using the fraction strategy, he revealed the following:

Q : What strategies did you use to overcome the problem?
M22 : I attempted to minimize the denominator.
Q : Why did you use that strategy?
M22 : I applied logical reasoning to anticipate the outcome of 1/2.
Q : What other strategies are you familiar with for solving comparison problems?
M22 : Most of the time, I utilize the cross-product method.

PMT worked by looking at half of what was known before but could not be precise in working with this fraction strategy. For example for stategy farcation $\frac{125}{10}$ $simplified$ $\frac{25}{2} = \frac{\ldots\times 25 = 300}{\ldots x\ 2 = \cdots.} = \frac{12\times 25}{12\ x\ 2.} = \frac{300}{24}$ so the result is 24 liter. While the fourth PMT teacher tried with Equation Strategy as a linear function y = kx in this solution pre-service mathematics teacher was unable to determine the x and y variables correctly so that he could not solve correctly. y = kx. The following is an interview with a PMT who answered with an equation strategy:

Q : What strategy did you use to solve the problem?
M13 : I used equations.
Q : Why did you choose that strategy?
M13 : I remembered the equations.
Q : What strategies are you familiar with for solving comparison problems?
M13 : cross-product, but I'm not sure if it involves cros-products on inverse proportion.

PMT uses an equation strategy because it looks at several questions that can be used as variables and one of the other variables is determined. Then our y variable represents the distance in km and the x variable represents the need for gasoline. $\frac{125}{10} = \frac{y}{x}$ then $125x = 10y$ as $12{,}5x = y$ so that when $y = 300$ then $12{,}5 \times \ldots = 300$ answer 24 liter. The fifth PST tried with the Building Up Strategy, namely trying to find a pattern of 125 km needed 10 liters, meaning that every 25 kg needed 2 liters, so the multiples were 125 km +125 km needed 10 liters + 10 liters. Then for 300 km = 125 +125 + 25(2) it takes 10 liters +10 liters + 4 liters. So, 300 kg requires 24 liters. The following interview with PMT explains that she uses the building up strategy, here are her expressions:

Q : What strategy did you use to solve the problem?
M1 : I'm not sure what it's called, Mom. I just looked at the question and recognized the pattern.
Q : Why did you choose to use that strategy?
M1 : It seemed like the best approach at the time.
Q : What strategies are you familiar with for solving comparison problems?"
M1 : I can't recall strategy any at the moment, Mom.

The Direct Proportion Design task can use the technique of a ratio table interpreted on a graph but this was not used by PMT. Pre-service mathematics teacher can create a ratio table from what is known and what is asked as in Table 3. The factor of change strategy leads students to find the multiplying factor which is then applied to determine the required value. Based on this, the solution to the problem presented in Figure 1 is to find the relationship of 125 km and 300 km. It turns out that 300 km $= \frac{12}{5} \times 125$ km. Meaning that125 is the multiplying factor, so that if 125 kg needs 10 liters of gasoline then for 300 km the requirement is $\frac{12}{5} \times 125$ km $= 24$ liter.

Based on the results of answers and interviews with PMT about solving inverse proportion tasks, it was revealed why PMT used various strategies. Most of the inverse proportion problems were solved using the cross-product method, which was based on learning experiences at school. However, most of the PMT experienced difficulties in answering inverse proportion problems because it was difficult to determine the ratio relationship properly, which resulted in errors in the final result. PMT often encounter difficulties in completing tasks related to inverse proportions, particularly in determining proportional relationships. Some PMT tend to believe that solving inverse proportion problems is the same as dealing with direct proportions, using cross-products without paying attention to the specific issues presented in the problem. As highlighted by Ölmez (2016), difficulties arise in forming proportional relationships and distinguishing between proportional and non-proportional relationships. According to (Arican, 2019a; Arican, 2020; Arican & Özçakir, 2021), determining and representing non-proportional relationships seems to be the most challenging task for pre-service mathematics teachers. When dealing with inverse proportion tasks, candidates face difficulties in finding mathematical formulas that can describe the relationship between the compared quantities.

These findings indicate a potential obstacle in their understanding of multiplication relationships, especially when involving more than two variables (Arican, 2018). These difficulties further complicate problem-solving through the use of proportional formulas, such as cross-multiplication strategies, without being able to interpret the words or answers of the problem correctly (Cabero-Fayos et al., 2020). Burgos et al. (2020) also demonstrate difficulties in understanding statement requirements, problem situation contexts, and mathematical procedures involved in solving tasks. In this regard, pre-service mathematics teachers tend to use more strategies to solve direct problems than inverse proportion problems (Arican et al., 2023).

Understanding the difficulties and strategies used by PMT in solving proportion tasks can serve as a foundation for developing a more effective and tailored learning trajectory to meet their needs. Targeting specific problem-solving hindrances can lead to the development of designs that enhance PMT competence in understanding proportion concepts. Furthermore, knowing the strategies employed by PMT in handling proportion tasks can assist curriculum developers in crafting more in-depth teaching materials. Integrating proven, effective learning strategies into curricula can aid teachers and future educators in facilitating a better understanding of future proportional concepts in the mathematics they will teach. Therefore, through a profound understanding of the difficulties and strategies of PMT in proportion tasks, we can enhance the effectiveness of mathematics education and produce more competent educators ready to face teaching challenges in the classroom.

Conclusion

This study concludes that there are five strategies used by PMTs in solving the proportion tasks. Among these strategies, PMT preferred the cross-product strategy due to its high efficiency. However, this approach often caused confusion, particularly regarding the ratio relationship in tasks involving inverse proportion. It is crucial for PMT to understand the significance of ratio relationships in such tasks. Specifically, they should start by forming two equivalent ratios in inverse proportion situations, then perform the cross-product procedure, and finally execute the division procedure. The failure to follow these steps can exacerbate confusion.

Moreover, it is important to acknowledge the limitations of this study. Further research is needed to explore the alternative instructional approaches to address the challenges faced by PMT in inverse proportion tasks. The limitations of this study encourage further research with a variety of proportion tasks to provide a more comprehensive understanding. It is also essential to consider the diverse learning styles of PMT in future studies to ensure that the strategies developed are effective for different types of learners. The educators are advised to integrate these findings into the development of hypothetical learning trajectories in the mathematics curriculum at Lembaga Pendidikan Tenaga Kependidikan (LPTK), especially in the Capita Selecta Mathematics courses for middle schools. This learning path can play an important role in equipping future teachers with a variety of strategies to effectively overcome proportion-related challenges in the classroom.

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