Beta: Jurnal Tadris Matematika, 17(1) 2024: 59-76
DOI 10.20414/betajtm.v17i1.624

Research article

Local instruction theory in the realistic mathematics education approach to improve students' mathematical proficiency in linear equation topics

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Abstrak Penelitian ini mengkaji peningkatan kemampuan matematis siswa pada materi persamaan garis lurus menggunakan *local instruction theory* (LIT) melalui pendekatan *realistic mathematics education* (RME). Penelitian ini menggunakan desain quasi-eksperimen, *one-group pretest-posttest*. Sampel penelitian adalah siswa SMP Negeri 1 Halmahera Selatan kelas VIII sebanyak 37 orang. Instrumen yang digunakan berupa tes kemampuan matematis yang terdiri atas 4 soal essai. Data dianalisis menggunakan uji prasyarat, uji Mann-Whitney, uji Cohens (d) *effect sizes*, dan juga dilakukan perhitungan nilai gain. Hasil penelitian menunjukkan bahwa terdapat perbedaan kemampuan matematis antara sebelum dan setelah penerapan LIT dengan pendekatan RME. Pengaruh LIT dengan pendekatan RME terhadap kemampuan matematis siswa berkategori sangat tinggi. Siswa mampu mendefinisikan persamaan garis lurus, mampu menentukan kemiringan suatu garis lurus, dapat menyusun persamaan garis lurus, mengintegrasikan suatu grafik garis lurus, dan menentukan solusi dari masalah yang berkaitan dengan persamaan garis lurus.

Kata kunci *Local instruction theory, Kemampuan matematis, Persamaan garis lurus, Realistic mathematics education*

Abstract The present study examines the improvement of students' mathematical proficiency in linear equation topics using local instruction theory (LIT) with a realistic mathematics education (RME) approach. This research employed a quasi-experiment, one-group pretest-posttest design. The sample was 37 secondary school students at SMP Negeri 1 South Halmahera, class VIII. The instrument used is a mathematical proficiency test consisting of 4 essay questions. Data were analyzed using the prerequisite test, Mann-Whitney test, Cohens (d) effect sizes test, and the gain value was determined. As a result, there are differences in students' mathematical proficiency before and after implementing LIT through the RME approach. The influence of LIT with the RME approach on mathematics proficiency is very high. The students can define a linear equation, determine the slope of the line, construct a linear equation, integrate the linear graphs, and determine solutions for the linear equation problems.

Keywords *Local instruction theory, Mathematical proficiency, Linear equation, Realistic mathematics education.*

Introduction

Mathematical proficiency is a students' ability to understand, apply and manipulate mathematical concepts effectively and efficiently. The importance of mathematical proficiency cannot be underestimated, because this skill forms the foundation for academic success in various fields of science as well as in everyday life (Cahyaningsih, 2023; National Research Council, 2001; Schoenfeld & Mathematical Sciences Research Institute, 2007). Mathematical

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proficiency includes five main components, i.e., conceptual understanding, procedural skills, strategic ability, reasoning skills, and productive disposition (Altarawneh & Marei, 2021; Li et al., 2021; Junpeng et al., 2020). Conceptual understanding allows students to see the relationships between different mathematical concepts, while procedural skills ensure that they can perform mathematical operations accurately and quickly. Strategic abilities and reasoning skills help students to solve complex problems using various appropriate strategies. A productive disposition encourages students to have a positive attitude towards mathematics, feel confident, and be persistent in solving problems (National Research Council, 2001).

However, the results of various international surveys such as PISA (Program for International Student Assessment) and TIMSS (Trends in International Mathematics and Science Study) show that, on average, the mathematical proficiency of the Indonesian students is generally still low. The data from PISA, for example, shows that many Indonesian students are below the international average in terms of mathematics ability (OECD, 2022). They often have difficulty in understanding basic mathematical concepts, performing basic mathematical operations, and applying their knowledge in new or unstructured situations. The students often memorize mathematical formula without really understanding its meaning, which evokes the obstacle when they are applying these concepts in different contexts (Wijayanto et al., 2024). Apart from that, the low ability to reason and solve mathematical problems is also an indication that the teaching methods used have not been able to develop proficiency holistically. As a consequence, the students are less prepared to face the academic challenges at higher levels of education and in their future professional lives.

The students are often struggling to understand the topics of linear equation and the way to apply it in real-world contexts because they learn mathematical subjects through memorization (Putri & Hidayati, 2022). For example, although students may be able to remember the equation of a linear is expressed in the form of $y = mx + c$, they often struggle to explain what m (gradient) and c (intercept) are or how to determine these values from graphs or data. This low ability shows that many students do not have adequate conceptual understanding. They also lack skill in the procedures necessary to solve problems involving linear equation, such as graphing an equation or determining the equality of two points (Dewi et al., 2021). Students' lack of strategic ability and reasoning skills can be seen from their inability to solve problems involving linear equation in various real situations, as for instance data trend analysis in economics, describing the relationship between two variables in physics (Muhlisin et al., 2022).

These facts are also supported by the results of preliminary observation, in which the students are still experiencing difficulties in learning linear equation concepts. One of the reasons is that because the teachers lack innovative teaching strategies, while only focusing on the written mathematical definitions in textbooks (Suryadi et al., 2023). The term “variable”, for instance, should be explained through various examples as well as its application in the real-life situations instead of merely telling the students its implicit definition based on what is written in the textbooks (Siagian et al., 2021). Another reason is that because the students lack understanding on prerequisite knowledges, including number patterns, sequence of number, cartesian coordinates, relation and function, while theoretically, the characteristics of students' experiences in studying mathematical contents in general are influenced by their learning experiences in previous subjects.

The low level of students’ mathematical proficiency in linear equations shows that the teaching methods currently used are not effective in developing in-depth understanding and practical application. The students are often involved in a learning scenario that is theoretical instead of being involved in the contextual learning environment that they are familiar with and relevant to everyday life. The realistic mathematics education (RME) approach is expected to be a solution for such learning problems. RME is an educational approach that emphasizes the importance of using real contexts and daily experiences in the mathematics learning process (Laurens et al., 2018; Da, 2023; Moleko & Mosimege, 2021). This approach aims to make mathematics more meaningful and relevant to the students, so that they would be able to connect the mathematical concepts being studied and the real situations they are experiencing every day. Given such a learning scenario, the students are expected to build a deeper and more intuitive understanding of mathematical concepts. In addition, the students would be actively engaged in the learning process through exploration, discussion, and reflection, which in turn can improve their ability to solve mathematical problems effectively (Sutarni & Aryuana, 2023). Putting mathematics in real and relevant contexts, the students would be able to comprehend both the mathematical contents and its application in real-life situations, which then creates more holistic and meaningful learning. Finally, RME teaching approach is expected to improve the quality of mathematics education which positively affects the students' competency, so they will be confident to overcome the mathematics challenges in the future.

According to the explanation above, it is necessary to develop a learning product which can be incorporated in the RME learning approach, referred to as Local Instruction Theory (LIT). LIT is an integral part of the Realistic Mathematics Education (RME) approach. LIT focuses on developing specific teaching theories for mathematical topics, which in this study is linear equations. The teachers can make a use of theory to design and implement the effective teaching strategies and relevant to real context (Das, 2020; Moleko & Mosimege, 2021). On top of that, the teachers can create a more meaningful learning experience for the students, where the concept of linear equation is integrated with the students' daily situations. The theory facilitates a deeper conceptual understanding as well as improves the students' ability to apply their mathematical knowledge to solve real-world problems. LIT provides a guideline and framework for teachers to develop teaching materials, plan learning activities, and evaluate students' understanding comprehensively (Ding et al., 2014; Nickerson & Whitacre, 2010; Cárcamo et al., 2019). Finally, LIT plays an important role in improving the quality of mathematics learning, especially in the topic of linear equation, through an approach that is more structured and relevant to the students' experiences.

The application of Local Instruction Theory (LIT) in the context of Realistic Mathematics Education (RME) is expected to improve students' mathematical proficiency. Through the LIT approach, the students are engaged to learn actively through exploration, discussion and in-depth reflection about the concept of linear equation (Hastuti & Fauzan, 2019). In the exploration process, they could discover and understand the concepts through various structured and contextual activities, which are directly related to real-life experiences and situations. The teacher-facilitated discussions allow students to share ideas, ask questions, and explore various ways of solving problems, thereby enriching their understanding of the mathematical contents. Through a deep reflection phase, the students could integrate new knowledge with the existing knowledge, as well as strengthen their conceptual understanding.

Several empirical studies have shown that the Realistic Mathematics Education (RME) approach along with the application of Local Instruction Theory (LIT) can significantly improve students' mathematics learning outcomes (Arini, 2019; Armiati et al., 2022; Ardiniawan et al., 2023; Supriatna et al., 2017). These studies indicate a marked improvement in the understanding of mathematical concepts, with students showing better ability to connect the theory with practical applications. Apart from that, the approach also improves students' problem-solving skills, making them more skilled in facing and solving various types of mathematical problems. The previous studies also revealed that the students who are being involved in the learning process with the integration of RME and LIT approaches tend to have more positive attitudes towards mathematics. They feel more confident and motivated in learning mathematics because the mathematical contents become more relevant and interesting to them. The learning process through RME and LIT thus significantly affects both students' cognitive and attitude aspects which are important factors for long-term education. Those empirical evidence support the effectiveness of RME and LIT in improving the quality of mathematics learning, which makes the approaches worth considering in mathematics education reform efforts.

The present study aims to improve students' mathematical proficiency through the implementation of Local Instruction Theory (LIT) in the Realistic Mathematics Education (RME) approach that focuses on linear equation. The research outcomes can be used to design the effective and relevant teaching strategies, and so that making a positive impact on the quality of mathematics education.

Theoretical review

Local instruction theory

LIT is an abbreviation of Local Instruction Theory. According to the Indonesian Big Dictionary (KBBI), local means occur in one place and are not evenly distributed in certain situations. Instruction is a guide to provide direction or guidance. The term "theory" is an experts' opinion, method, and rules for an activity. Thus, LIT is a method to guide students in learning a particular topic/material with a detailed, gradual, and specific scope, tools, or learning media. LIT is a framework for achieving learning objectives by considering students' mathematical preparation assumptions (Nickerson & Whitacre, 2010; Bustang et al., 2013; Prahmana & Suwasti, 2014).

The LIT construct in teaching shows the teachers' frame of reference in designing learning with teaching materials that focus on concepts by considering student difficulties, anticipating all possible student responses, and making mathematics learning more meaningful. LIT is a theory that can guide someone to learn about a particular topic or material. Therefore, LIT is a reference that can improve the quality of the learning trajectory. LIT is closely related to learning objectives, learning activities, and the role of learning tools and media (Prahmana & Suwasti, 2014; Wijayanti et al., 2022).

According to Prahmana (2017), the final product of the Hypothetical learning trajectory (HLT) that has been designed, implemented, and analyzed for learning outcomes is LIT. where HLT is a learning process design that pays attention to student responses. "HLT is a hypothesis or prediction of how students' thinking and understanding develop in learning activities. The learning flow in the hypothetical is an assumption about the series of activities that the child goes through in solving a problem or understanding a concept.

LIT is a theory that explains the learning process, which describes the learning trajectory on a particular topic with a set of supporting activities. LIT only discusses a specific topic in an individual lesson. To design LIT, what must be considered are: 1) the sequence must be an authentic experience, 2) students must be guided to rediscover concepts, and 3) develop design models from teaching materials.

Realistic mathematics education

A method of teaching mathematics known as realistic mathematics education (RME) connects the mathematical concepts to real-world situations to help the students towards a better understanding of the mathematical knowledge and so that they would be able to apply mathematics to their own lives. The students are encouraged to think about discovering problems, organizing the subject matter, and solving difficulties as a source for the development of formal mathematical concepts or knowledge in realistic problems (Das, 2020; Altaylar & Kazak, 2021; Tong et al., 2022).

Freudenthal created RME for the first time in 1971 at the Dutch Utrecht University. Freudenthal asserts that mathematics learning is an activity, hence the classroom is not a place for teachers to impart knowledge to their students but rather is a setting for students to rediscover mathematical ideas and concepts through investigation of actual problems (Revina & Leung, 2021; Bayrak & Aslanci, 2022).

According to Gravemeijer (in Nugraheni & Marsigit, 2021), the RME approach follows three guiding concepts that aid pupils in learning mathematics: (1) Progressive mathematization and guided reinvention, i.e., students should have the chance to apply the same mathematical ideas they are taught through the courses covered. Didactic Phenomenology (2), i.e., the presentation of mathematical themes is based on two factors, namely their use and contribution to the advancement of other mathematical ideas. (3) Models created by oneself, i.e., the purpose of self-developed models is to help students move from hypothetical to concrete situations or from informal to formal mathematics, allowing them to solve issues on their own.

The RME also has its own characteristics. The characteristics of the RME approach that can be applied for learning mathematics are expected to help students in solving mathematical problems. The characteristics of RME are expressed by Treffers (Hafidah & Rukli, 2022), namely: (1) Using the real world. Mathematics learning does not start with a formal system but begins with contextual problems (the real world). In this case, students use previous experience directly. (2) Using examples, i.e., the term "model" refers to mathematical and situational models created by pupils on their own (self-developed models). Self-developed models have the function of bridging students from tangible to abstract situations or from informal to formal situations. (3) Employing manufacturing and construction, i.e., the chance for students to create informal problem-solving techniques that can result in the creation of solving procedures exists. With regard to production and building, students are urged to consider the elements they feel crucial to the learning process. Students are expected to be able to formally reinvent mathematical ideas with the teacher's supervision. (4) Using Interactive Interaction between students and teachers is very basic in the process of learning realistic mathematics. (5) Intertwining study units, i.e., in realistic mathematical learning, mathematical units in the form of learning phenomena are interrelated and very necessary. This linkage will facilitate students' process of solving problems.

Mathematical proficiency

Mathematical proficiency is a skill that corresponds to expertise, competence, knowledge, and skills in mathematics to determine success in mathematics and in other fields (Retnawati & Wulandari, 2019; Junpeng et al., 2020). This is in line with Kilpatrick's opinion (in Corrêa & Haslam, 2020), which states that mathematical proficiency is used to explain aspects of expertise, competence, knowledge, and facilities in mathematics so that it becomes successful. Mathematical proficiency has five interrelated components, namely: 1) conceptual understanding; 2) procedural fluency; 3) strategic competence; 4) adaptive reasoning; and 5) productive disposition. The five components form strands and support each other, as shown in the following Figure 1.

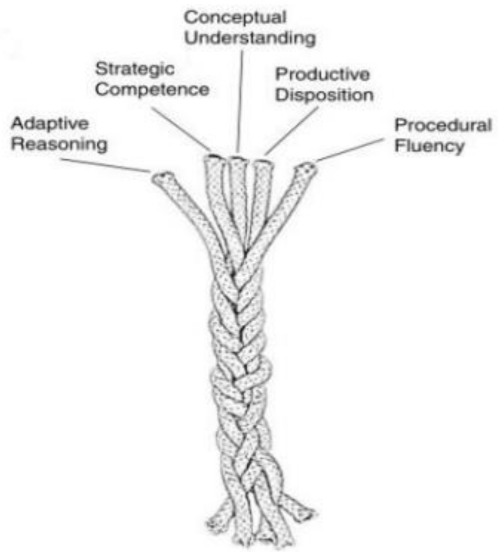

Figure 1. Intertwined strands of mathematical proficiency

The link between the five components of mathematical proficiency above is described as follows: Comprehension of mathematical notions, procedures, and relationships is referred to as conceptual comprehension. Students can only become proficient in arithmetic if they have a solid conceptual understanding of the subject. Two skills that are indicators of conceptual comprehension are the capacity to comprehend various visuals and the ability to describe mathematical situations in various ways.

The ability to apply steps in a flexible, precise, and effective manner is known as procedural fluency. Students who lack conceptual comprehension abilities but possess procedural fluency tend to memorize things without understanding what they are doing, which leads to incorrect usage of processes. Procedural fluency indications include the following: 1) the ability to recall, choose, and apply the appropriate formula; 2) accuracy in computations; and 3) accuracy in the use of algorithms.

The capacity for problem-solving in mathematics is referred to as strategic competency. Students have fewer options for problem-solving solutions due to a lack of strategic competency without conceptual comprehension and procedural fluency. Indicators of strategic competence include the following: 1) the ability to formulate problems; 2) planning the appropriate strategy;

3) effectively representing the situation; 4) carrying out the planned strategy; and 5) resolving issues using the appropriate strategy.

The capacity to reflect, explain, and justify oneself is known as adaptive reasoning, which involves thinking rationally between contexts and mathematical concepts. Students having adaptive reasoning skills without conceptual knowledge, procedural fluency, and strategic competence will not be able to solve problems because they lack the necessary conceptual understanding and strategic competence. Making conjectures or cojectures, arguing that a proposition is true, drawing conclusions, determining whether an argument is legitimate, determining patterns in the information being taught, and so forth are all examples of adaptive reasoning.

A productive disposition, namely a mathematical thought that is useful and valuable, sees oneself as an effective learner. Beliefs, attitudes, and confidence generally serve as indicators of a creative temperament. With regard to the relationship between the other four components and the productive disposition, which is the belief that mathematics is understandable with efforts and can be learned, teachers must pay attention to the following: (1) Mathematics learning is focusing and coherently emphasizing skills on main topics such as integers, fractions, geometry, and measurement. (2) Developing conceptual understanding, procedural fluency, and problem-solving skills at the same time. (3) Using a variety of approaches to meet the needs of students (4) Using formative assessment regularly to assess student learning (Phaniew et al., 2021).

Mathematical proficiency is the ability to understand, use, and even formulate a definition. Mathematical proficiency can also wholly describe all aspects of skills, competencies, knowledge, and facilities in mathematics and is necessary for anyone to learn mathematics correctly.

Methods

Research design

The research uses a quantitative method with a quasi-experimental one group pretest-posttest design (see Figure 2). Before implementing the local instruction theory (LIT) in RME approach, the students were given a pretest in the form of a mathematical proficiency test. Next, the treatment process was conducted by implementing the local instruction theory (LIT) in the RME approach for teaching and learning linear equation topics. The number of face-to-face meetings with students was 5 meetings, with the same number of hours according to the number of hours regulated in the mathematics curriculum in junior high school. After applying local instruction theory (LIT) in the RME approach, the researcher gave a final test, namely the posttest.

O	*X*	*O*
Pretest	Treatment	*Posttest*

Figure 2. One-group pretest-posttest design (Creswell, 2015)

Information
O: Pretest and Posttest
X: local instruction theory in the RME approach

Population and sample

The research population were junior high school students in South Halmahera Regency, Bacan City, Indonesia. The sample were 37 students grade VIII at secondary school of SMP Negeri 1 South Halmahera, which were selected through purposive sampling technique.

Research instruments

The test instrument was used to measure students' mathematical proficiency. The items were arranged in such a way that these were relevant and in line to the lesson objectives and linear equations contents.

Table 1. Test instrument for mathematical proficiency

<table>
<tr><th>No.</th><th>Question</th></tr>
<tr><td>1</td><td>a) Given the line equation $y = x - 2$, complete the following table.
<table>
<tr><td>x</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>y</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>(x, y)</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
According to the table in question a),
b) Draw a graph $y(x)$.
c) Determine the value of x, when $y = 5$.</td></tr>
<tr><td>2</td><td>a) Given the coordinate of A (-3, 6) and B (7, -4), determine the line equation connecting point A and point B.
b) Determine the gradient of line in a.</td></tr>
<tr><td>3</td><td>Look at the following table, which shows the relationship between the time used to inflate the tire and the resulting air pressure in the tire.
<table>
<tr><td>Time (seconds)</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>Pressure (psi)</td><td>2</td><td>6</td><td>10</td><td>14</td><td>18</td><td>22</td></tr>
</table>

a) Draw a graph that explains the relationship between pumping time and air pressure in the tire.
b) Determine the gradient and the linear equation that describes the situation.</td></tr>
<tr><td>4</td><td>Show that the line $7x + 2y = 10$ is parallel to a line $4y - 9x + 12 = 0$.</td></tr>
</table>

The test items are essay-type questions as seen in Table 1, taking into account the indicators of mathematical proficiency. The test items were assessed by the experts in terms of face and content validity as well as the reliability aspect. The Q-Cochran test was carried out to see the uniformity of the judgement outcomes. Next, the instrument evaluation was continued to the readability testing on several state junior high school students in South Halmahera. The students were given the test items to see whether these can be understood properly or not. The next level of validity is empirical validity, where the test instrument was given to a group of students in a representative school with a research sample (Samura & Habsyi, 2023).

The outcomes of the experts' judgements to the face and content validity of the test items are displayed in Table 2.

Table 2. Summary of face and content validity of test items

Validity test	Statistics		
	Q-Cochran	*Df*	*Asymp.sig.*
Face validity	2.33	2	0.31
Content validity	3.00	2	0.22

As seen in Table 2 the sig value for both types of validity are more significant than 0.05. This means that the three validators are uniform in providing assessments to the test items. Hence, the conclusion can be drawn that both face and content validity of the test are fulfilled. The reliability value is 0.81, meaning that the test instrument is highly reliable and consistent.

Data analysis techniques

With the aid of many statistical tests, the obtained data were quantitatively examined. The data homogeneity and normality checks were carried out first, followed by the statistical test. As for the stages of data processing, before data processing was carried out, data completeness was checked first (following the data collection). Checking the quality of the data was carried out at this stage by observing whether the answers from students are as expected by the researcher, in the sense that all questions are answered satisfactorily. Checking the quality of the data was done by ascertaining the missing data to decide if it is necessary to search for additional data. Once all the data has been declared complete, the next step is calculation. Calculations were made in the form of:

a. Pretest, Posttest, and Gain Scores Descriptive Statistics Calculation, including Minimum, Maximum, Average, and Standard Deviation Scores without establishing generalizations about the population, descriptive statistical analysis only presents an overview of the sample.
b. Calculating the increase in students' mathematical proficiency using the Normalized Gain formula developed by Hake, the N-Gain formula is:

$$NG = \frac{Posttest\ Score - Pretest\ Score}{Ideal\ Maximum\ Score - Pretest\ Score}$$

The Gain index criteria is shown in Table 3, (Samura & Darhim, 2023).

Table 3. Criteria for normalized gain scores

Normalized gain score (g)	Interpretation
$g \geq 0.70$	High
$0.30 \leq g < 0.70$	Medium
$g < 0.30$	Low

c. Calculating the effectiveness of local instruction theory for linear equation subject in the RME learning scenario by using effect sizes developed by Cohens' d with the formula:

$$Cohens'\ d = \frac{\bar{d}}{Sd}$$

The classification of Cohens' (d) effect sizes can be shown in Table 4 (Samura & Habsyi, 2023).

Table 4. Cohens' (d) effect size classification

Effect size	Criteria
$0.00 \leq ES < 0.20$	Very small
$0.20 \leq ES < 0.50$	Small
$0.50 \leq ES < 0.80$	Currently
$0.80 \leq ES < 1.30$	High
$1.30 \leq ES$	Very Big

d. Inferential statistical analysis was used for data analysis.

Findings and Discussion

The findings of this study were obtained from the results of quantitative data processing originating from the results of the pretest and posttest related to mathematical proficiency. Descriptively, the pretest, posttest and N-gain mathematical proficiency are presented in Table 5.

Table 5. Students' mathematical proficiency scores

Value	N	Min	Max	Mean	Std. Deviation
Pretest	37	5	37	17.59	8.325
Posttest	37	60	95	78.57	9.55
N-Gain	37	0.49	0.94	0.741	0.11433

Table 5 shows that the pretest results before the intervention were carried out. Students' mathematical proficiency scores had a fairly wide range with a minimum score of 5 and a maximum score of 37. The average pretest score was 17.59, which shows that, in general, students' mathematical abilities are still at a good level. low. The fairly large standard deviation (8.325) indicates significant variation in students' mathematical proficiency before the intervention. The posttest results showed a significant increase in students' mathematical proficiency scores after the intervention. The minimum score increased to 60, and the maximum score reached 95. The average posttest score was 78.57, indicating an increase in students' overall mathematical proficiency. The standard deviation of 9.55 indicates that although there is a significant improvement, there is still variation in students' abilities after the intervention. Still, this variation is relatively smaller compared to the pretest results.

In general, the data in Table 5 shows that the intervention successfully increased students' mathematical proficiency significantly. The average posttest score is much higher than the pretest, and the high N-Gain indicates that the approach used effectively improves students' understanding and mathematical skills. Smaller variations in scores after intervention also indicate more even improvements among students. Furthermore, statistical tests were carried out to strengthen the justification for the results of descriptive analysis regarding the increase in students' mathematical proficiency before and after implementing LIT with the RME approach. Table 6 shows the results of the pretest, posttest, and N-Gain data normality tests, which are statistical prerequisite tests.

Table 6. Results of normality test of students' mathematical proficiency data

Value	Statistic test			Conclusion
	N	S-W	*Sig.*	
Pretest	37	0.918	0.01	Reject H_0
Posttest	37	0.971	0.437	Accept H_0
N-Gain	37	0.982	0.811	Accept H_0

Table 6 shows the results of the normality test on the pretest, posttest and N-Gain scores. The results of the Shapiro-Wilk (S-W) test show that the pretest data are not normally distributed (sig. < 0.05), while the posttest data (sig. > 0.05) and N-Gain (sig. > 0.05) show a normal distribution. Based on the results of the normality test, it was found that there were data scores that were not normal, so the prerequisite test in the form of homogeneity was not necessary.

The next stage is to conduct a two-mean test after the necessary tests. Based on the results of the prerequisite test above, it is hereby decided to test the difference between the two means using nonparametric statistics using the Mann-Whitney test. The test criterion in the Mann-Whitney test is that if the Asymp.Sig (2-tailed) value is less than alpha (0.05), then there is a significant difference; in other circumstances, there is no significant difference. The following statistical hypothesis is put forward as follows:

H_0 : There is no difference in learning outcomes before and after implementing LIT with the RME approach.

H_1 : There is difference in learning outcomes before and after implementing LIT with the RME approach.

The Mann-Whitney test table, which is presented below, explains the completeness of the statistical test findings on the provided hypothesis. To clarify, a summary of the Mann-Whitney test results is presented in Table 7.

Table 7. Differences in mathematical proficiency as measured by mann-whitney test

Learing	Statistical test			
	Mann-whitney	Wilcoxon W	Z	Asymp.sig (2-tailed)
LIT with RME approach	268	934	-4.400	0.000

The results of the Mann-Whitney test in Table 7 show a significant difference in mathematical proficiency between before and after the intervention (Asymp. Sig < 0.05). The Z value is negative (-4.400), and the Asymp value is. A very small sig. indicates that the increase in students' Mathematics Proficiency after the intervention is statistically significant. In other words, applying LIT with the RME approach affects students' mathematical proficiency.

The following is to determine the effectiveness of applying LIT with the RME approach to students' mathematical proficiency using the Cohen (d) effect size quantity. Cohens' (d) effect sizes can be calculated based on the information displayed in Table 5. As a result, Table 8 summarizes the outcomes of the Cohen (d) effect size estimation.

Table 8. Results of the cohens' (d) effect sizes test

Learning	Effect size	Criteria
LIT with RME Approach	6.806971	Very High

As seen in Table 8, the Cohens' d effect size of 6.806971 shows that the learning intervention greatly increases students' mathematical proficiency. The "Very High" criterion indicates that the intervention implemented effectively improves students' mathematical proficiency. N-Gain (Normalized Gain) is used to measure the effectiveness of the intervention by comparing the increase in scores from pretest to posttest. The average N-Gain of 0.741 in Table 5 shows a relatively high increase in students' mathematical proficiency scores. N-Gain's minimum score of 0.49 and maximum of 0.94 (in Table 5) indicates that all students experienced an increase in their scores, although at varying levels. The relatively small standard deviation (0.11433) indicates that the score increase was fairly consistent among students.

Local Instruction Theory (LIT) is important in structuring specific and contextual learning. By using LIT, teachers can design and implement teaching strategies that are more effective and relevant to students' daily experiences. In the context of learning linear equation, LIT helps teachers make the material more meaningful and exciting for students by using real contexts that students can understand and relate to their lives.

The Realistic Mathematics Education (RME) approach supports LIT by emphasizing real situations and everyday experiences as a context for understanding mathematical concepts. Through RME, students are invited to actively participate in the learning process through exploration, discussion, and reflection. Students can understand the idea better in the linear equation material because they see how it is applied in real situations, such as measuring distances, calculating speeds, or determining positions.

The results of data analysis show that before the intervention, students' mathematical proficiency scores were, on average, 17.59, with a standard deviation of 8.325. This score shows that students' mathematical abilities in the linear equation are low. After implementing LIT with the RME approach, students' posttest scores increased significantly to an average of 78.57 with a standard deviation of 9.55. This significant increase is also supported by an average N-Gain value of 0.741, which indicates a high increase in students' mathematical abilities.

The results of the Mann-Whitney test confirmed that the difference in scores before and after the intervention was statistically significant, with a Z value of -4.400 and an Asymp.Sig (2-tailed) value of 0.000. This shows that the improvements that occurred were not coincidental but the result of effective intervention. In addition, Cohen's d effect size of 6.806971, which is in the very high category, strengthens the finding that applying LIT with the RME approach has a large impact on increasing students' mathematical proficiency.

Local Instruction Theory (LIT) with a Realistic Mathematics Education (RME) approach helps students understand the linear equation material by making learning more contextual, explorative, collaborative, and reflective. Using real context, students can see the relevance of mathematics in everyday life, while in-depth exploration and discussion help them develop a solid conceptual understanding (Ling & Mahmud, 2023; Purwasih et al., 2024). This approach also improves students' problem-solving skills and makes learning more meaningful. The two pictures below (Figure 3 and Figure 4) show how students' mathematical proficiency differs before and after being given the intervention.

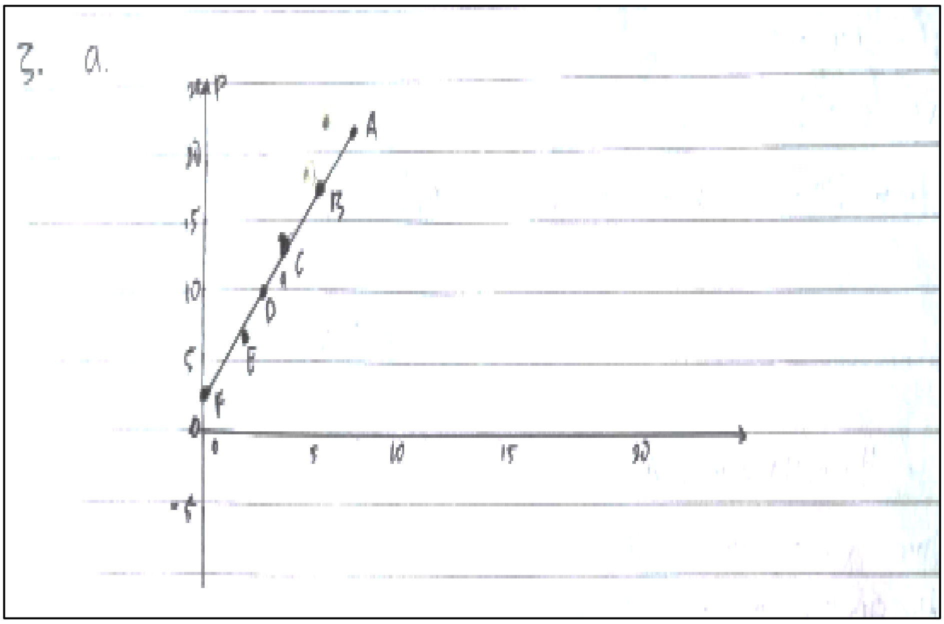

Figure 3. Student answers prior to the intervention

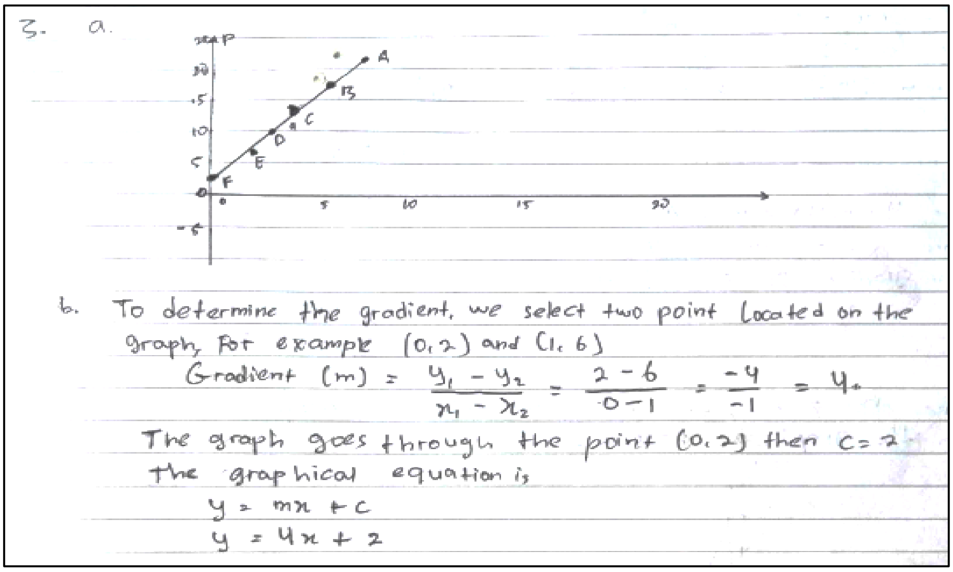

Figure 4. Student answers after intervention

The students' answers in Figure 3 and Figure 4 answer the question number 3 where data points are known, which shows the relationship between the time used to inflate the tire and the air pressure produced in the tire. The students are asked to (1) draw a graph that explains the relationship between pumping time and air pressure in the tire, (2) determine the gradient and the linear equation that describes the situation.

The students' answers before the intervention (Figure 3) show that students' mathematical proficiency is still lacking. In contrast, the results show that students can only draw graphs related to the situation from the data provided. Students' mathematical proficiency was better after the intervention (Figure 4). Aspects of students' mathematical proficiency related to (1) conceptual understanding, (2) procedural fluency, (3) strategic competence, (4) adaptive reasoning, and (5) productive disposition are presented correctly and systematically. Conceptual understanding and procedural fluency are demonstrated by students being able to draw a line equation that explains the relationship between the pumping time and the air pressure in the tire. Furthermore, students demonstrate strategic competence and adaptive reasoning by being able to use and evaluate the gradient of the line that has been drawn. In determining the general equation of a line, students can determine arbitrary values for constants based on arbitrary points (0.2) so that c = 2 (strategic competence, adaptive reasoning, and productive disposition).

Increasing students' mathematical proficiency after applying LIT with the RME approach impacts students' ability to master the concept of linear equation. This combination also encourages students to think critically and analytically and develop problem-solving skills that are essential in mathematics. By integrating theory and practice, students memorize formulas and understand how and why these formulas work (Mokhtar et al., 2023; Angraini et al., 2023). This makes learning more meaningful and students more motivated to learn.

This research supports the findings of various previous studies, which show the effectiveness of the RME and LIT approaches in improving students' mathematical abilities. Like the research of Çakıroğlu et al. (2023), by expanding RME, they found an increase in students' mathematical literacy abilities. The results of this research show an increase in the moderate category in all dimensions of mathematical literacy abilities: employing, interpreting, and formulating, and at the bottom, interpreting dimension. The study conducted by Ventistas et al. (2024) demonstrated the positive impact of RME interventions on students' problem-solving skills and discussed how short-term interventions can influence students' performance in large-scale international assessments. Similar results were also found in Şanal and Elmali's (2023) research; their findings revealed that RME had a significant and positive influence on students' mathematical problem-solving performance. In addition, a study by Bakker and van Eerde (2015) showed that LIT is effective in developing specific and relevant teaching strategies, which ultimately improve student learning outcomes. This suggests that local teaching theories can help teachers by developing viable local teaching theories that can be used by classroom teachers to interpret hypothetical learning trajectories that suit students' actual characteristics and situations. Gravemeijer (2004) explained that, in line with the RME theory that inspired the design of LIT, this allows teachers to design an instruction that helps students develop their current ways of thinking into more sophisticated mathematical ways of thinking. Based on the findings of this study and relevant research findings, it shows that RME not only improves students' conceptual understanding, but also increases their positive attitudes towards mathematics. In line with the findings of this study, it shows that students not only experienced an increase in test scores, but also showed increased interest and motivation in learning mathematics.

Conclusion

To summarize, the implementation of Local Instruction Theory (LIT) in the Realistic Mathematics Education (RME) was effective in increasing students' mathematical proficiency in the linear equation. The research outcomes show a significant increase in students' mathematics scores after being intervened, both in terms of conceptual understanding, procedural skills, and the ability to apply mathematics in real situations. Furthermore, it improves the students' mathematical proficiency towards high category. The objectives of learning linear equations were achieved, including the ability to define linear equation, determine the gradient or slope of a linear, construct linear equation, integrate linear graphs, and determine solutions to problems related to linear equation. The results of this study are consistent with previous research findings, which strengthen the validity and reliability of this approach in the context of mathematics education. The learning scenario designed in this study indeed improves the students' mathematical abilities and makes the learning process becomes more relevant and exciting. Further research is needed to explore the application of LIT and RME to other mathematical contents and to examine the long-term sustainability of these positive effects.

Acknowledgment

The researcher would like to thank the Ministry of Religion of the Republic of Indonesia through the Litapdimas program, which has provided research cost using the Ternate State Islamic Institute DIPA in 2023 for data collection thereby the researcher can prepare research reports properly.

References

Altarawneh, A. F., & Marei, S. T. (2021). Mathematical proficiency and preservice classroom teachers' instructional performance. *International Journal of Education and Practice*, *9*(2), 354–364. https://doi.org/10.18488/journal.61.2021.92.354.364

Altaylar, B., & Kazak, S. (2021). The effect of realistic mathematics education on sixth grade students' statistical thinking. *Acta Didactica Napocensia*, *14*(1), 76–90. https://doi.org/10.24193/adn.14.1.6

Angraini, L. M., Larsari, V. N., Muhammad, I., & Kania, N. (2023). Generalizations and analogical reasoning of junior high school viewed from Bruner's learning theory. *Infinity*, *12*(2), 291–306. https://doi.org/10.22460/infinity.v12i2.p291-306

Ardiniawan, D. Y., Subiyantoro, S., & Kurniawan, S. B. (2023). Effectiveness of the RME (realistic mathematical education) approach to learning achievement in view of students' mathematic reasoning. *QALAMUNA: Jurnal Pendidikan, Sosial, dan Agama*, *14*(2), 783-800. https://doi.org/10.37680/qalamuna.v14i2.3520

Arini, A. (2019). Development of local instruction theory of multiplication based on realistic mathematics education in primary schools. *International Journal of Educational Dynamics*, *1*(1), 188–204. https://doi.org/10.24036/ijeds.v1i1.54

Armiati, A., Fauzan, A., Harisman, Y., & Sya'bani, F. (2022). Local instructional theory of probability topics based on realistic mathematics education for eight-grade students. *Journal on Mathematics Education*, *13*(4), 703–722. https://doi.org/10.22342/jme.v13i4.pp703-722

Bakker, A., van Eerde, D. (2015). An Introduction to design-based research with an example from statistics education. In: Bikner-Ahsbahs, A., Knipping, C., Presmeg, N. (eds) *Approaches to Qualitative Research in Mathematics Education. Advances in Mathematics Education*. Springer, Dordrecht. https://doi.org/10.1007/978-94-017-9181-6_16

Bayrak, A., & Aslanci, S. (2022). Realistic mathematics education: A bibliometric analysis. *International Journal of Education*, *10*(4), 52–62. https://doi.org/10.1007/978-94-007-4978-8_170

Bustang, B., Zulkardi, Z., Darmawijoyo, D., Dolk, M., & van Eerde, D. (2013). Developing a local instruction theory for learning the concept of angle through visual field activities and spatial representations. *International Education Studies*, *6*(8), 58–70. https://doi.org/10.5539/ies.v6n8p58

Cahyaningsih, U., Jatisunda, M.G., Kurniawan, D.T., Nahdi, D.S., Utami, W.P., & Halipah, R. (2023). Implementing problem-based learning to enhance students' mathematical proficiency in primary school. *Jurnal Didaktik Matematika*, *10*(2), 281–299. https://doi.org/10.24815/jdm.v10i2.32615

Çakıroğlu, Ü., Güler, M., Dündar, M., & Coşkun, F. (2023). Virtual reality in realistic mathematics education to develop mathematical literacy skills. *International Journal of Human–Computer Interaction*, 1–13. https://doi.org/10.1080/10447318.2023.2219960

Cárcamo, A., Fuentealba, C., & Garzón, D. (2019). Local instruction theories at the university level: An example in a linear algebra course. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(12), em1781. https://doi.org/10.29333/ejmste/108648

Corrêa, P. D., & Haslam, D. (2020). Mathematical proficiency as the basis for assessment: A literature review and its potentialities. *Mathematics Teaching-Research Journal*, *12*(4), 3–20. https://commons.hostos.cuny.edu/mtrj/wp-content/uploads/sites/30/2021/01/v12n4-Mathematical-Proficiency-as-the-Basis-for-Assessment.pdf

Creswell, J. (2015). *Educational research, planning, conducting, and evaluating quantitative and qualitative (5th ed.)*. Pearson Education, Inc.

Da, N. T. (2023). Realistic mathematics education and authentic learning: A combination of teaching mathematics in high schools. *Journal of Mathematics and Science Teacher*, *3*(1), em029. https://doi.org/10.29333/mathsciteacher/13061

Das, K. (2020). Realistic mathematics & Vygotsky's theories in mathematics education. *Shanlax International Journal of Education*, *9*(1), 104–108. https://doi.org/10.34293/education.v9i1.3346

Dewi, N., Talib, A., & Djam'an, N. (2021). Student difficulties in learning mathematics based on learning styles. *Advances in Social Science, Education and Humanities Research*, *611*, 11–18. https://www.atlantis-press.com/proceedings/icoesm-21/125965695

Ding, L., Jones, K., Pepin, B., & Sikko, S. A. (2014). An expert teacher's local instruction theory underlying a lesson design study through school-based professional development. *North American Chapter of the Psychology of Mathematics Education*, *2*, 401–408. https://files.eric.ed.gov/fulltext/ED599772.pdf

Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning*, *6*(2), 105–128. https://doi.org/10.1207/s15327833mtl0602_3

Hafidah, H., & Rukli, R. (2022). Treatment slow learner learning repetitive addition with realistic mathematics learning approach. *Mimbar Sekolah Dasar*, *9*(3), 396–412. https://doi.org/10.53400/mimbar-sd.v9i3.48586

Hastuti, E., & Fauzan, A. (2019). Penerapan local instructional theory menggunakan pendekatan RME untuk meningkatkan hasil belajar siswa di sekolah dasar. *Jurnal Basicedu*, *3*(2), 271–276. https://doi.org/10.31004/basicedu.v3i2.2

Junpeng, P., Marwiang, M., Chiajunthuk, S., Suwannatrai, P., Chanayota, K., Pongboriboon, K., Tang, K. N., & Wilson, M. (2020). Validation of a digital tool for diagnosing mathematical proficiency. *International Journal of Evaluation and Research in Education*, *9*(3), 665–674. https://doi.org/10.11591/ijere.v9i3.20503

Laurens, T., Batlolona, F. A., Batlolona, J. R., & Leasa, M. (2018). How does realistic mathematics education (RME) improve students' mathematics cognitive achievement?. *Eurasia Journal of Mathematics, Science and Technology Education*, *14*(2), 569-578. https://doi.org/10.12973/ejmste/76959

Li, Y., Howe, R. E., Lewis, W. J., & Madden, J. J. (2021). Developing mathematical proficiency for elementary instruction: An introduction. In Y Li, R. E. Howe, W. J. Lewis, & J. J. Madden (Eds.), *Developing mathematical proficiency for elementary instruction. Advances in STEM education* (pp. 3–9). Springer, Cham. https://doi.org/10.1007/978-3-030-68956-8_1

Ling, A. N. B., & Mahmud, M. S. (2023). Challenges of teachers when teaching sentence-based mathematics problem-solving skills. *Frontiers in Psychology*, *13*, 1074202. https://doi.org/10.3389/fpsyg.2022.1074202

Mokhtar, N., Xuan, L. Z., Lokman, H. F., & Mat, N. H. C. (2023). Theory, literature review, and fun learning method effectiveness in teaching and learning. *International Journal of Social Science and Education Research Studies*, *3*(8), 1738–1744. https://doi.org/10.55677/ijssers/V03I8Y2023-30

Moleko, M. M., & Mosimege, M. D. (2021). Flexible teaching of mathematics word problems through multiple means of representation. *Pythagoras*, *42*(1), 1–10. https://doi.org/10.4102/pythagoras.v42i1.575

Muhlisin, A., Sarwanti, S., Jalunggono, G., Yusliwidaka, A., Mazid, S., & Mohtar, L. E. (2022). Improving students' problem-solving skills through RIAS model in science classes. *Jurnal Cakrawala Pendidikan*, *41*(1), 284–294. https://doi.org/10.21831/cp.v41i1.47263

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education,

Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.
Nickerson, S. D., & Whitacre, I. (2010). A local instruction theory for the development of number sense. *Mathematical Thinking and Learning*, *12*(3), 227–252. https://doi.org/10.1080/10986061003689618
Nugraheni, L. P., & Marsigit, M. (2021). Realistic mathematics education: An approach to improve problem solving ability in primary school. *Journal of Education and Learning*, *15*(4), 511–518. https://doi.org/10.11591/edulearn.v15i4.19354
OECD. (2022). *PISA 2022 mathematics framework*. Paris: OECD Publishing.
Phaniew, S., Junpeng, P., & Tang, K. N. (2021). Designing standards-setting for levels of mathematical proficiency in measurement and geometry: Multidimensional item response model. *Journal of Education and Learning*, *10*(6), 103–111. https://doi.org/10.5539/jel.v10n6p103
Prahmana, R. C. I. (2017). *Design research (teori dan implementasinya: Suatu pengantar)*. Jakarta: Rajawali Pers.
Prahmana, R. C. I., & Suwasti, P. (2014). Local instruction theory on division in mathematics gasing: The case of rural area's student in Indonesia. *Journal on Mathematics Education*, *5*(1), 17–26. http://dx.doi.org/10.22342/jme.5.1.1445.17-26
Purwasih, R., Turmudi, & Dahlan, J. A. (2024). How do you solve number pattern problems through mathematical semiotics analysis and computational thinking?. *Journal on Mathematics Education*, *15*(2), 403–430. http://doi.org/10.22342/jme.v15i2.pp403-430
Putri, D. R., & Hidayati, N. (2022). Mathematical problem solving ability of junior high school students on linear equations. *Unnes Journal of Mathematics Education Research*, *11*(2), 6–11. Retrieved from https://journal.unnes.ac.id/sju/ujmer/article/view/67166
Retnawati, H., & Wulandari, N. F. (2019). The development of students' mathematical literacy proficiency. *Problems of Education in the 21st Century*, *77*(4), 502–514. https://doi.org/10.33225/pec/19.77.502
Revina, S., & Leung, F. K. S. (2021). Issues involved in the adoption of realistic mathematics education in Indonesian culture. *Compare*, *51*(5), 631–650. https://doi.org/10.1080/03057925.2019.1650636
Samura, A. O., & Darhim. (2023). Improving mathematics critical thinking skills of junior high school students using blended learning model (BLM) in geogebra assisted mathematics learning. *International Journal of Interactive Mobile Technologies*, *17*(02), 101–117. https://doi.org/10.3991/ijim.v17i02.36097
Samura, A. O., & Habsyi, R. (2023). Application of the blended learning model to improve the mathematical creative thinking skills of geogebra-assisted junior high school students in mathematics lessons. *European Journal of Mathematics and Science Education*, *4*(2), 149–159. https://doi.org/10.12973/ejmse.4.2.149
Şanal, S. Ö., & Elmali, F. (2023). Effectiveness of realistic math education on mathematical problem-solving skills of students with learning disability. *European Journal of Special Needs Education*, *39*(1), 109–126. https://doi.org/10.1080/08856257.2023.2191110
Schoenfeld, A. H., & Mathematical Sciences Research Institute. (2007). What is mathematical proficiency and how can it be assessed? In A. H. Schoenfeld (Ed.), *Assessing Mathematical Proficiency* (pp. 59–74). chapter, Cambridge: Cambridge University Press.
Siagian, M. D., Suryadi, D., Nurlaelah, E., Tamur, M., & Sulastri, R. (2021). Investigating students' concept image in understanding variables. *Journal of Physics: Conference Series*, *1882*(1), 012058. http://doi.org/10.1088/1742-6596/1882/1/012058
Supriatna, T., Darhim, D., & Turmudi, T. (2017). Local intruction theory dalam pendidikan matematika realistik untuk menumbuhkan kemampuan berpikir logis. *MIMBAR PENDIDIKAN: Jurnal Indonesia untuk Kajian Pendidikan*, *2*(2), 173–184. https://doi.org/10.17509/mimbardik.v2i2.8627
Suryadi, D., Itoh, T., & Isnarto. (2023). A prospective mathematics teacher's lesson planning: An in-depth analysis from the anthropological theory of the didactic. *Journal on Mathematics Education*, *14*(4), 723–740. http://doi.org/10.22342/jme.v14i4.pp723-740
Sutarni, S., & Aryuana, A. (2023). Realistic mathematics education (RME): Implementation of learning models for improving HOTS-oriented mathematics problem-solving ability. *AL-ISHLAH: Jurnal Pendidikan*, *15*(2), 1213–1223. https://doi.org/10.35445/alishlah.v15i2.2127
Tong, D. H., Nguyen, T. T., Uyen, B. P., Ngan, L. K., Khanh, L. T., & Tinh, P. T. (2022). Realistic mathematics education's effect on students' performance and attitudes: A case of ellipse topics

learning. *European Journal of Educational Research*, *11*(1), 403–421. https://doi.org/10.12973/eu-jer.11.1.403

Ventistas, G., Ventista, O. M., & Tsani, P. (2024). The impact of realistic mathematics education on secondary school students' problem-solving skills: A comparative evaluation study. *Research in Mathematics Education*, 1–25. https://doi.org/10.1080/14794802.2024.2306633

Wijayanti, D. A., Sampoerno, P. D., & Hajizah, M. N. (2022). The local instructional theory on introducing concept of functions. *ITALIENISCH*, *12*(2), 1153–1160.

Wijayanto, P. W., Lumbantoruan, J. H., Judijanto, L., Mardikawati, B., & Huda, N. (2024). Mathematics teachers in teaching 4C skills: School-university perspective. *Journal of Education Research and Evaluation*, *8*(1), 195–206. https://doi.org/10.23887/jere.v8i1.67895